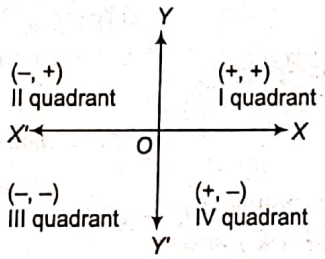
CHAPTER

15

Coordinate System and Straight Lines

Coordinate System

A system in which points are defined as the ordered pair (x, y), is called the coordinate system. Here, x and y are called abscissa and ordinate respectively.



In coordinate system, there are two perpendicular straight lines XOX' and YOY' called axes.
The axes divide the coordinate plane is four quadrants. Therefore, OX and OX' are called positive and negative directions respectively of x-axis and OY and OY' are called positive and negative directions respectively of y-axis

Distance Formula

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points in the xy-plane, then

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or $|PQ| = \sqrt{(\text{difference of abscissas})^2 + (\text{difference of ordinates})^2}$

Example 1. Find a point P which is equidistant from the three points $A(0, 1)$, $B(1, 0)$ and $C(4, 3)$.

(a) (1, 2) (b) (2, 2)
(c) (2, 3) (d) (3, 3)

Sol. (b) Let $P(a, b)$ be the required point. We are given that $PA = PB = PC$ or $PA^2 = PB^2 = PC^2$.

Now, $PA^2 = PB^2$

We get, $a^2 + (b-1)^2 = (a-1)^2 + b^2$

$$a^2 + b^2 - 2b + 1 = a^2 - 2a + 1 + b^2$$

$$\Rightarrow \quad b = a$$

and $PA^2 = PC^2$, we get

$$a^2 + (b-1)^2 = (a-4)^2 + (b-3)^2$$

$$\Rightarrow \quad a^2 + b^2 - 2b + 1 = a^2 - 8a + 16 + b^2 - 6b + 9$$

$$\Rightarrow \quad 8a + 4b - 24 = 0$$

Substituting $b = a$, we get

$$12a - 24 = 0$$

$$\Rightarrow \quad a = 2$$

Hence, the required point is $P(2, 2)$.

- In order to prove that a given figure is a square, prove that the four sides are equal and the diagonals are also equal.
- Rhombus, prove that the four sides are equal.
- Rectangle, prove that the opposite side are equal and the diagonals are also equal.
- A parallelogram, prove that the opposite sides are equal.
- Parallelogram but not a rectangle, prove that its opposite sides are equal but the diagonals are not equal.

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- A Rhombus but not a square, prove that its all sides are equal but the diagonals are not equal.
- For three points to be collinear, prove that the sum of the distances between two pairs of points is equal to the third pair of points.
- In a parallelogram, diagonals cut each other at mid-point.

Example 2. The points $A(1, 2)$, $B(2, 5)$ and $C(5, 6)$ are the vertices of

(a) an equilateral triangle
(b) an isosceles triangle
(c) a scalene triangle
(d) a right angled isosceles triangle

Sol. (b) $AB^2 = (2-1)^2 + (5-2)^2 = 10$

$\Rightarrow \quad AB = \sqrt{10}$

$AC^2 = (5-1)^2 + (6-2)^2 = 32$

$\Rightarrow \quad AC = \sqrt{32}$

$BC^2 = (5-2)^2 + (6-5)^2$

$= 10$

$\Rightarrow \quad BC = \sqrt{10}$

Since, $AB = BC = \sqrt{10}$, the given points form an isosceles triangle.

But $AC^2 \neq AB^2 + BC^2 \quad (\because 32 \neq 10 + 10)$

$\therefore$ It is not right angled.

Example 3. Under which one of the following conditions, the coordinates $(0, 0), (h, k), (h + h', k + k')$ and (h', k'), taken in order will be the vertices of a parallelogram?

(a) $hh' + kk' = 0$
(b) $hh' - kk' = 0$
(c) It is true for all values of h, h', k and k'
(d) $h = h', k = k'$

Sol. (c) The coordinates $A(0,0), B(h,k), C(h+h', k+k')$ and $D(h', k')$ taken in order will be the vertices of a parallelogram, if mid-point of AC = mid-point of BD

i.e., $\left(\frac{h+h'+0}{2}, \frac{k+k'+0}{2}\right) = \left(\frac{h+h'}{2}, \frac{k+k'}{2}\right)$

$\Rightarrow \quad \left(\frac{h+h'}{2}, \frac{k+k'}{2}\right) = \left(\frac{h+h'}{2}, \frac{k+k'}{2}\right)$

Which is true for all values of h, h', k and k'.

Area of a Triangle

Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a ΔABC, then

Area of ΔABC

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Condition of Collinearity of Three Points

The three points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear, if they lie on a straight line. *i.e.*, if the area of ΔABC is zero. Hence, the condition that the points A, B and C are collinear is

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

Example 4. If the points with the coordinates $(a, ma), \{b, (m+1)b\}, \{c, (m+2)c\}$ are collinear, then which one of the following is correct?

(a) a, b, c are in AP for all m
(b) a, b, c are in GP for all m
(c) a, b, c are in HP for all m
(d) a, b, c are in AP only for $m = 1$

Sol. (c) Since, the points $A(a, ma), B[b, (m+1)b]$ and $C[c, (m+2)c]$ are collinear.

$\therefore \quad a\{(m+1)b - (m+2)c\} + b\{(m+2)c - ma\}$

$+ c\{ma - (m+1)b\} = 0$

$\Rightarrow \quad mab + ab - mac - 2ac + mbc + 2bc$

$- mab + mac - mbc - bc = 0$

$\Rightarrow \quad ab - 2ac + 2bc - bc = 0$

$\Rightarrow \quad ab + bc = 2ac$

$\Rightarrow \quad b = \frac{2ac}{a+c}$

$\Rightarrow a, b, c$ are in HP for all m.

Section Formulas

(i) **Point of internal division** Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the two given points. Let the point $R(x, y)$ divide and the line joining the points P and Q internally in the ratio $m : n$, then

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

(ii) **Point of external division** If R divides the points P and Q externally in the ratio $m : n$, then

$$(x, y) = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

(iii) **Mid-point** If R divides the points P and Q internally in the ratio $1 : 1$. *i.e.*, If R is the mid-point of PQ, then

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

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Example 5. The coordinates of P and Q are $(-3, 4)$ and $(2, 1)$, respectively. If PQ is extended to R such that $PR = 2QR$, then what are the coordinates of R?

(a) $(3, 7)$ (b) $(2, 4)$ (c) $\left(-\frac{1}{2}, \frac{5}{2}\right)$ (d) $(7, -2)$

Sol. (d) Since, the coordinates of P and Q are $(-3, 4)$ and $(2, 1)$ respectively.

Let coordinates of R are (x, y).

Also, $PR = 2QR \Rightarrow m : n = 2 : 1$

$$\therefore \quad x = \frac{2\times 2 + 1\times 3}{2-1} = 4 + 3 = 7 \text{ and } y = \frac{2\times 1 - 1\times 4}{2-1} = 2 - 4 = -2$$

$\therefore$ Coordinates of R are $(7, -2)$.

Example 6. In what ratio does the line $y - x + 2 = 0$ cut the line joining $(3, -1)$ and $(8, 9)$?

(a) $2 : 3$ (b) $3 : 2$
(c) $3 : -2$ (d) $1 : 2$

Sol. (a) Let C be the point which divides the join of $(3, -1)$ and $(8, 9)$ in the ratio $\lambda : 1$.

Then the coordinates of C are

$$\left(\frac{\lambda\times 8 + 1\times 3}{\lambda+1}, \frac{9\times\lambda - 1\times 1}{\lambda+1}\right) = \left(\frac{8\lambda+3}{\lambda+1}, \frac{9\lambda-1}{\lambda+1}\right)$$

This point also lies on $y - x + 2 = 0$

$$\therefore \quad \frac{9\lambda-1}{\lambda+1} - \frac{8\lambda+3}{\lambda+1} + 2 = 0$$

$$\Rightarrow \quad 9\lambda - 1 - 8\lambda - 3 + 2\lambda + 2 = 0 \Rightarrow 3\lambda - 2 = 0 \Rightarrow \lambda = 2/3$$

$\therefore$ Required ratio $= 2 : 3$

Some Important Points of a Triangle

Let ABC be a triangle with vertices $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$.

Centroid of a Triangle

The centroid of a triangle is the point of intersection of its medians.

The coordinates of the centroid G of a triangle whose vertices are $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Incentre of a Triangle

The incentre of a triangle is the point of intersection of the internal bisectors of the angles.

If (x, y) be the incentre of the ΔABC, where $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ and a, b, c are the sides of the triangle, then

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Circumcentre of a Triangle

The circumcentre of a triangle is the point of intersection of the perpendicular bisectors of its sides.

Also, it is the centre of the circle passing through the vertices of the triangle.

The coordinates of the circumcentre of a triangle whose vertices are $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$$\left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}\right)$$

If we don't know about the values of the angles A, B and C and if $P(x, y)$ be the circumcentre of ΔABC, then

$$PA = PB = PC \qquad \ldots(i)$$

We can determine the coordinates of P by simplifying Eq. (i) for x and y.

Orthocentre of a Triangle

The orthocentre of a triangle is the point of intersection of its altitudes.

The coordinates of the orthocentre of a triangle whose vertices are $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$$\left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C}\right)$$

If we don't know about the values of the angles A, B and C, then in order to find the coordinates of the orthocentre of a triangle, we first find the equations of its altitudes, then we find the coordinates of the point of intersection of any two of them.

- If the vertices of a triangle have integral coordinates, (or rational) then the triangle can't be equilateral.
- If the direction of the two sides of a triangle is fixed and length of the third side is constant and is sliding between these sides, then locus of the orthocentre of the triangle is circle.
- Orthocentre of a triangle will lie inside, if triangle is acute and in case of obtuse triangle, orthocentre will lie out side and in case of right triangle orthocentre is the vertex at which it is right angled.
- In the normal form of a straight line the sum of the squares of the coefficient of x and y is equal to one.
- Three or more straight lines are said to be concurrent lines, if they meet in a point.

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Example 7. The vertices of a triangle are $A(5,-1)$, $B(-1,5)$ and $C(6,6)$. Find the coordinates of the circumcentre.

(a) $\left(\frac{10}{3},\frac{10}{3}\right)$ (b) $\left(\frac{23}{8},\frac{23}{8}\right)$

(c) $\left(\frac{28}{3},\frac{28}{3}\right)$ (d) None of these

Sol. (b) Let $P(x,y)$ be the circumcentre of the ΔABC. We have,

$$PA = PB = PC \text{ or } PA^2 = PB^2 = PC^2$$

Now, $PA^2 = PB^2$, we get

$$(x-5)^2+(y+1)^2=(x+1)^2+(y-5)^2$$

$$\Rightarrow \quad x^2+y^2-10x+2y+26=x^2+y^2+2x-10y+26$$

$$\Rightarrow \quad x-y=0 \qquad \ldots(i)$$

Now, $PB^2 = PC^2$, we get

$$(x+1)^2+(y-5)^2=(x-6)^2+(y-6)^2$$

$$\Rightarrow \quad x^2+y^2+2x-10y+26=x^2+y^2-12x-12y+72$$

$$\Rightarrow \quad 7x+y-23=0 \qquad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$x=23/8 \text{ and } y=23/8$$

Hence, the coordinates of the circumcentre are (23/8, 23/8).

Locus of a Point

The curve described by a point which moves under given conditions is called its locus.

Equation of Locus

It is an equation in x and y, which is satisfied by every point on the locus and by no point outside the locus.

Working Rule to Find the Locus of a Point

(i) Let $P(h,k)$ be any point on the locus.

(ii) Write the equation in mathematical form involving h, k.

(iii) Eliminate the variables if any.

(iv) Replace h by x and k by y in the result obtained in the equation so obtained is the locus of the point which moves under some equation given conditions.

Example 8. The sum of the squares of the distances of a moving point from two fixed points $(p,0)$ and $(-p,0)$ is equal to a constant quantity $2\lambda^2$, find the equation of the locus.

(a) $x^2+y^2=\lambda^2-p^2$

(b) $x^2+y^2=\lambda^2+p^2$

(c) $x^2-y^2=\lambda^2-p^2$

(d) $x^2-y^2=\lambda^2+p^2$

Sol. (a) Let $P(h,k)$ be any position of the moving point and $A(p,0), B(-p,0)$ be the given points. Then,

$$PA^2+PB^2=2\lambda^2$$

$$\Rightarrow \quad (h-p)^2+(k-0)^2+(h+p)^2+(k-0)^2=2\lambda^2$$

$$\Rightarrow \quad h^2-2ph+p^2+k^2+h^2+2ph+p^2+k^2=2\lambda^2$$

$$\Rightarrow \quad 2h^2+2k^2+2p^2=2\lambda^2$$

$$\Rightarrow \quad h^2+k^2=\lambda^2-p^2$$

Hence, locus of (h,k) is $x^2+y^2=\lambda^2-p^2$.

Straight Line

The equation of a straight line l is a linear relationship in x and y *i.e.*, it is a first degree polynomial equation in x and y.
Equation of the form $ax + by + c = 0$
where a, b and c are constants, represents a straight line in general form.

Equations of Line in Different Forms

(i) **Slope-intercept form** $y=mx+c$ is called the slope form, slope-intercept form or the tangent form of the equation of the line c is called the intercept made by the line on the y-axis.

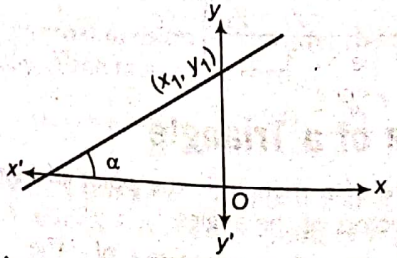
To find the slope and the intercept on the y-axis of a line given in the form $ax+by+c=0$, we write it in the form

$$y=-\frac{a}{b}x-\frac{c}{b}$$

Therefore, the slope of the line is $m=-a/b$ and the intercept of the line on the y-axis is $-c/b$.

(ii) **Slope-point form** The equation of a line passing through the point (x_1, y_1) and having the slope 'm' is

$$y-y_1=m(x-x_1)$$



(iii) **Two point form** The equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is

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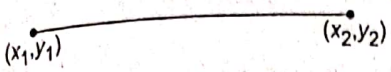
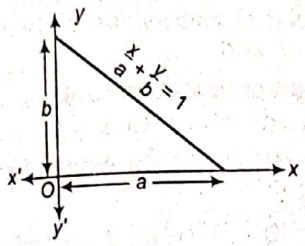
$$(y-y_1)=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

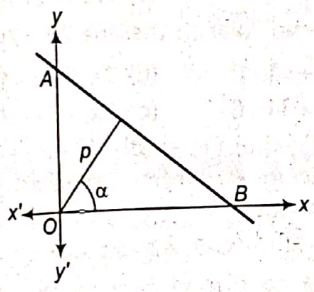


(iv) **Intercept form** If a line cuts the intercepts 'a' and 'b' on the axes, then the equation of line is

$$\frac{x}{a}+\frac{y}{b}=1$$



(v) **Normal form** If p is the normal from the origin to the line AB and if the normal makes an angle α with x-axis, then the equation of the line in the form $x\cos\alpha+y\sin\alpha=p$ is called the normal form of the line.



Slope or Gradient of a Line

The trigonometrical tangent of the angle 'α' which a given line makes with the positive direction of x-axis, is called the slope or gradient of the line. It is generally denoted by m.

i.e. $$m=\tan\alpha$$

Slopes in Different Conditions

(i) Slope of x-axis or a line parallel to x-axis is

$$m=\tan 0^\circ=0$$

(ii) Slope of y-axis or a line parallel to y-axis is

$$m=\tan\frac{\pi}{2}=\infty$$

(iii) The slope of the line joining the points (x_1,y_1) and (x_2,y_2) is obtained as

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{\text{difference of ordinates}}{\text{difference of abscissae}}$$

Angle Between Two Lines

Angle θ between two lines $y=m_1x+c_1$ and $y=m_2x+c_2$ is given by

$$\tan\theta=\pm\frac{m_2-m_1}{1+m_1m_2}$$

Let two lines AB and CD be given in general form, *i.e.*,

equation of AB is $a_1x+b_1y+c_1=0$ and

equation of CD is $a_2x+b_2y+c_2=0$

Then, slope of $AB=m_1=-a_1/b_1$

and slope of $CD=m_2=-a_2/b_2$

The angle between the lines becomes

$$\tan\theta=\left|\frac{-(a_2/b_2)+(a_1/b_1)}{1+(a_1a_2)/(b_1b_2)}\right|=\left|\frac{a_1b_2-a_2b_1}{a_1a_2+b_1b_2}\right|$$

Above lines are parallel, if

$$a_1b_2-b_1a_2=0 \quad \text{or} \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}$$

The lines are perpendicular, if $a_1a_2+b_1b_2=0$

Some Facts About General Equation of Line $ax+by+c=$

(i) The slope of the line $ax+by+c=0$ is

$$m=-\frac{a}{b}=-\frac{\text{coefficient of } x}{\text{coefficient of } y}$$

(ii) The intercepts of the line $ax+by+c=0$ on the x-axis and the y-axis are $(-c/a)$ and $(-c/b)$ respectively.

(iii) Let two straight lines be given by

$$a_1x+b_1y+c_1=0$$

and $$a_2x+b_2y+c_2=0$$

Then, the lines are

(a) coincident or identical, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(b) parallel, if their slopes are equal, *i.e.*,

$$m_1=m_2 \text{ or } -\frac{a_1}{b_1}=-\frac{a_2}{b_2}$$

or $$\frac{a_1}{b_1}=\frac{a_2}{b_2}$$

(c) perpendicular, if the product of their slopes is -1, *i.e.*,

$$m_1m_2=-1$$

or $$\left(-\frac{a_1}{b_1}\right)\left(-\frac{a_2}{b_2}\right)=-1$$

or $$a_1a_2+b_1b_2=0$$

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(iv) The equation of a line which is

(a) parallel to $ax+by+c=0$ can be written as

$$ax+by+k=0 \qquad (\because m_1=m_2)$$

(b) perpendicular to $ax+by+c=0$ can be written as

$$bx-ay+k=0 \qquad (\because m_1m_2=-1)$$

where k is an arbitrary constant to be determined using a given condition.

Example 9. What is the acute angle between the lines $Ax+By=A+B$ and $A(x-y)+B(x+y)=2B$?

(a) 45° (b) $\tan^{-1}\left(\frac{A}{\sqrt{A^2+B^2}}\right)$

(c) $\tan^{-1}\left(\frac{B}{\sqrt{A^2+B^2}}\right)$ (d) 60°

Sol. (a) The given equations of lines are

$$Ax+By=A+B \qquad ...(i)$$

and $A(x-y)+B(x-y)=2B$

$\Rightarrow$ $(A+B)x+(B-A)y=2B$ ...(ii)

Here, $a_1=A, b_1=B, c_1=A+B$

$a_2=A+B, b_2=B-A$

and $c_2=2B$

$$\therefore \quad \theta=\tan^{-1}\left|\frac{a_2b_1-a_1b_2}{a_1a_2+b_1b_2}\right|$$

$$=\tan^{-1}\left|\frac{AB+B^2-AB+A^2}{A^2+AB+B^2-AB}\right|=\tan^{-1}\left|\frac{A^2+B^2}{A^2+B^2}\right|$$

$$=\tan^{-1}(1)$$

$$=45°$$

Example 10. Find the equation of the line joining the points (3, −1) and (2, 3). Also, find the equation of another line perpendicular to this line and passing through the point (5, 2).

(a) $4x+y-11=0, x-4y+3=0$

(b) $4x+y+11=0, x-4y+3=0$

(c) $x+4y-11=0, 4x-y+3=0$

(d) $x-4y+11=0, 4x-y+3=0$

Sol. (a) Equation of line passing through (3, −1) and (2, 3)

$$y+1=\frac{3+1}{2-3}(x-3) \Rightarrow y+1=-4(x-3)$$

$\Rightarrow$ $4x+y-11=0$

which is the required equation.

Equation of line perpendicular to it

$$x-4y+k=0$$

But it passes through the point (5, 2). So, it will satisfy this equation.

$\therefore$ $5-4(2)+k=0 \Rightarrow k=3$

Hence, the another required equation is

$$x-4y+3=0$$

Family of Lines

Let $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ be two given lines.

Then, the equation

$$a_1x+b_1y+c_1+\lambda(a_2x+b_2y+c_2)=0 \qquad ...(i)$$

represents the equation of a line, passing through the point of intersection of the given lines for any arbitrary value of λ.

Example 11. Find the equation of the line which passes through the point of intersection of the lines $x-2y+3=0$ and $2x+y-4=0$ and

(I) passes through the point (3, 4)

(a) $x+y+5=0$ (b) $x+y+3=0$

(c) $x+y+1=0$ (d) None of these

(II) has slope 2

(a) $x+y+2=0$ (b) $2x-y=0$

(c) $2x+y=0$ (d) $x+2y+2=0$

(III) is parallel to the line $3x+5y-1=0$

(a) $3x+5y+1=0$ (b) $3x+y+13=0$

(c) $3x+5y+13=0$ (d) None of these

(IV) is perpendicular to the line $2x-7y+2=0$

(a) $2x-7y+11=0$ (b) $7x+2y-11=0$

(c) $7x+2y+11=0$ (d) $2x+7y-11=0$

Sol. (I) → (d), (II) → (b), (III) → (d), (IV) → (b)

The equation of the line which passes through the point of intersection of the lines

$$x-2y+3=0 \text{ and } 2x+y-4=0$$

can be written as

$$(x-2y+3)+\lambda(2x+y-4)=0$$

$\Rightarrow$ $(1+2\lambda)x+(-2+\lambda)y+3-4\lambda=0$ ...(i)

where λ is an arbitrary constant to be determined.

(I) Since, the line (i) passes through the point (3, 4), we have

$$(1+2\lambda)3+(-2+\lambda)4+3-4\lambda=0$$

$\Rightarrow$ $6\lambda-2=0 \Rightarrow \lambda=1/3$

Therefore, equation of the required line is

$$\left(1+\frac{2}{3}\right)x+\left(-2+\frac{1}{3}\right)y+3-\frac{4}{3}=0$$

$\Rightarrow$ $\frac{1}{3}(5x-5y+5)=0 \Rightarrow x-y+1=0$

(II) Slope of the line (i) is

$$m=\frac{-(1+2\lambda)}{-2+\lambda}=2$$

$\Rightarrow$ $-1-2\lambda=-4+2\lambda \Rightarrow 4\lambda=3 \Rightarrow \lambda=3/4$

The equation of the required line becomes

$$\left(1+\frac{3}{2}\right)x+\left(-2+\frac{3}{4}\right)y+3-3=0$$

$\Rightarrow$ $\frac{5}{2}x-\frac{5}{4}y=0 \Rightarrow 2x-y=0$

(III) The line (i) is parallel to the line $3x+5y-1=0$

Therefore, their slopes are equal. Hence,

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$$\frac{-(1+2\lambda)}{-2+\lambda}=-\frac{3}{5}\Rightarrow 5+10\lambda=-6+3\lambda$$

$$7\lambda=-11\Rightarrow \lambda=-11/7$$

$\Rightarrow$ The equation of the line becomes

$$\left(1-\frac{22}{7}\right)x+\left(-2-\frac{11}{7}\right)y+3+\frac{44}{7}=0$$

$$\Rightarrow \quad -\frac{15}{7}x-\frac{25}{7}y+\frac{65}{7}=0$$

$$\Rightarrow \quad 3x+5y-13=0$$

(iv) The line (i) is perpendicular to the line $2x-7y+2=0$. Therefore, the product of their slopes must be equal to -1. We get,

$$\left(-\frac{1+2\lambda}{-2+\lambda}\right)\left(\frac{2}{7}\right)=-1$$

$$\Rightarrow \quad 2+4\lambda=-14+7\lambda\Rightarrow 3\lambda=16\Rightarrow \lambda=16/3$$

Thus equation of the line becomes

$$\left(1+\frac{32}{3}\right)x+\left(-2+\frac{16}{3}\right)y+3-\frac{64}{3}=0$$

$$\Rightarrow \quad \frac{1}{3}(35x+10y-55)=0$$

$$\Rightarrow \quad 7x+2y-11=0$$

Concurrency of Three Lines

To find, whether three lines are concurrent or not, we find the point of intersection of any two lines. If this point lies on the third line, *i.e.*, the coordinates of this point of intersection satisfy the equation of the third line, then the straight lines are concurrent.

The condition that the three lines

$$a_1x+b_1y+c_1=0 \quad ...(i)$$

$$a_2x+b_2y+c_2=0 \quad ...(ii)$$

and

$$a_3x+b_3y+c_3=0 \quad ...(iii)$$

are concurrent, if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

Example 12. If the lines $3x-4y-13=0$, $8x-11y-33=0$ and $2x-3y+\lambda=0$ are concurrent, then value of λ is

(a) 7 (b) -7

(c) $\frac{1}{7}$ (d) $-\frac{1}{7}$

Sol. (b) Given lines are concurrent, then

$$\begin{vmatrix} 3 & -4 & -13 \\ 8 & -11 & -33 \\ 2 & -3 & \lambda \end{vmatrix}=0$$

$$\Rightarrow \quad 3(-11\lambda-99)+4(8\lambda+66)-13(-24+22)=0$$

$$\Rightarrow \quad -\lambda-7=0\Rightarrow \lambda=-7$$

Perpendicular Distance of a Point From a Line

The perpendicular distance d from $P(x_1,y_1)$ to the line is given by $ax+by+c=0$ is

$$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

(i) The perpendicular distance from the origin to the line $ax+by+c=0$ is

$$d=\left|\frac{c}{\sqrt{a^2+b^2}}\right|$$

(ii) Consider now, the two parallel lines

$$ax+by+c_1=0 \quad \text{and} \quad ax+by+c_2=0$$

Let OP and OP' be the perpendicular distances from the origin to the given lines respectively. Then, the distance between the parallel lines is given by

$$d=|OP-OP'|=\left|\frac{c_1}{\sqrt{a^2+b^2}}-\frac{c_2}{\sqrt{a^2+b^2}}\right|$$

or $$d=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$$

Example 13. The intercepts of a straight line upon the coordinate axes are a and b. If the length of the perpendicular on this line from the origin be 1, then which one of the following relations is correct?

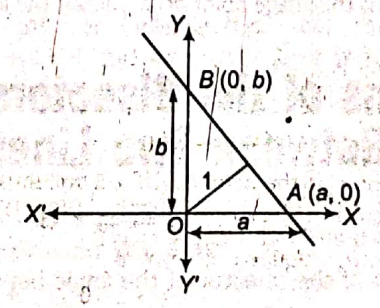
(a) $\frac{1}{a^2}+\frac{1}{b^2}=2$ (b) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{\sqrt{2}}$

(c) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{2}$ (d) $\frac{1}{a^2}+\frac{1}{b^2}=1$

Sol. (d) The equation of line passing through A and B is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \quad \frac{x}{a}+\frac{y}{b}-1=0 \quad ...(i)$$



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Length of perpendicular from (0, 0) to the Eq. (i)

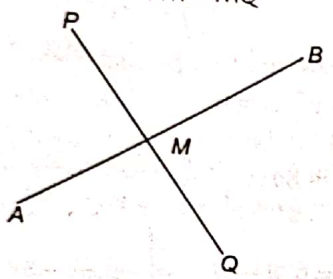
$$= \left| \frac{\frac{0}{a} + \frac{0}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right|$$

$$\therefore \quad 1 = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = 1$$

Image of a Point About a Straight Line

Let AB be a given line and P be any point in the plane. Draw PM perpendicular to AB. Extend PM to Q such that

$$PM = MQ$$



Then, the point Q is called the image of the point P about the straight line AB. Where M is the mid-point of PQ.

Example 14. Find the image of the point $P(2, 1)$ about the straight line $x - y + 5 = 0$.

(a) $(-4, 7)$ (b) $(4, 7)$
(c) $(4, -7)$ (d) $(-4, -7)$

Sol. (a) The slope of the given line is 1. Therefore, slope of the perpendicular line from P to the line is -1. Hence, the equation of the line PM, passing through $P(2, 1)$ and having slope -1, is

$$y - 1 = -1(x - 2) \quad \text{or} \quad x + y - 3 = 0$$

Solving the equations $x - y + 5 = 0$ and $x + y - 3 = 0$, we obtain the coordinates of the point M as $(-1, 4)$.

Let the coordinates of the point Q be (h, k). Since, Q is the image of P, M is the mid-point of PQ. We get,

$$(h + 2)/2 = -1$$

and $$(k + 1)/2 = 4,$$

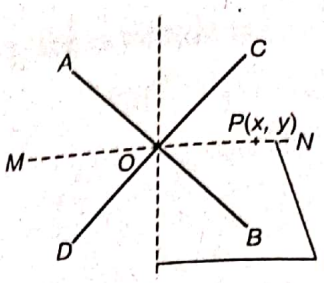
$$\Rightarrow \quad h = -4 \text{ and } k = 7$$

Therefore, $(-4, 7)$ is the image point of $(2, 1)$ about the line $x - y + 5 = 0$.

Equations of the Bisectors of Angles Between Two Lines

Let AB and CD be two given lines represented by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

Then, equation of the bisector of the angle between them are



Bisectors of the angles between AB and CD

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Example 15. Find the equations of the bisector of the angles between the straight lines $3x + 4y + 7 = 0$ and $12x + 5y - 8 = 0$

(a) $7x + 9y + 17 = 0, 99x + 77y + 51 = 0$
(b) $7x - 9y - 17 = 0, 99x + 77y - 51 = 0$
(c) $7x - 9y + 17 = 0, 99x + 77y + 51 = 0$
(d) None of the above

Sol. (d) Equations of the bisectors of the angles between the given straight lines are given by

$$\frac{3x + 4y + 7}{\sqrt{9 + 16}} = \pm \frac{12x + 5y - 8}{\sqrt{144 + 25}}$$

$$\Rightarrow \quad 13(3x + 4y + 7) = \pm 5(12x + 5y - 8)$$

$$\Rightarrow \quad 39x + 52y + 91 = \pm(60x + 25y - 40)$$

Taking positive sign,

$$39x + 52y + 91 = 60x + 25y - 40$$

$$\Rightarrow \quad -21x + 27y + 51 = 0$$

$$\Rightarrow \quad 7x - 9y - 17 = 0$$

Taking negative sign,

$$(39x + 52y + 91) = -(60x + 25y - 40)$$

$$\Rightarrow \quad 99x + 77y + 51 = 0$$

- If (x_1, y_1) and (x_2, y_2) are the end of hypotenuse of a right angled isosceles triangle, then the third vertex is given by
$$\left(\frac{x_1 + x_2 \pm (y_1 - y_2)}{2}, \frac{y_1 + y_2 \pm (x_1 - x_2)}{2} \right)$$
- Given two vertices (x_1, y_1) and (x_2, y_2) of an equilateral triangle, then its third vertex is given by
$$\left(\frac{x_1 + x_2 \pm \sqrt{3}(y_1 - y_2)}{2}, \frac{y_1 + y_2 \mp \sqrt{3}(x_1 - x_2)}{2} \right)$$
- Orthocentre of the right angled ΔABC, right angled at A is A.
- x-axis divides the line segment joining (x_1, y_1) and (x_2, y_2) in the ratio $-y_1 : y_2$ and y-axis divides the same line segment in the ratio $-x_1 : x_2$.
- If ABCD is a parallelogram, then $D = A - B + C$.
- If D, E, F are the mid-points of sides BC, CA, AB of a ΔABC, then the centroid of ΔABC = centroid of ΔDEF.

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- Orthocentre, centroid, circumcentre of triangle are collinear, centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.
- The circumcentre of a right angle triangle is the mid-point of the hypotenuse.
- In an equilateral triangle orthocentre, centroid, circumcentre, incentre coincide.
- A triangle is isosceles if any two of its medians are equal.
- The foot of the perpendicular (h, k) from (x_1, y_1) to the line $ax + by + c = 0$ is given by

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$$

- The image or (reflection) of the point (x_1, y_1) in the line $ax + by + c = 0$ is (h, k)

where $$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

- Area of rhombus formed by $ax \pm by \pm c = 0$ is $\left|\frac{2c^2}{ab}\right|$.
- Area of parallelogram formed by the lines

$$a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0,$$
$$a_1x + b_1y + d_1 = 0,\ a_2x + b_2y + d_2 = 0,$$

is $\left|\frac{(d_1 - c_1)(d_2 - c_2)}{a_1b_2 - a_2b_1}\right|$

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