14

# HCF and LCM of Polynomials

## HCF of Polynomials

The highest common factor of two or more algebraic expressions is the factor of the highest degree which divides each of the expressions without a remainder.

It is denoted by HCF or GCD.

- It is a convention to take HCF in such a way that the coefficient of the highest power of the variable is positive.

## HCF of Monomials

- To find the HCF of two or more monomials, we multiply the HCF of the numerical coefficients of the monomials by the highest power of each of the letters common to both the polynomials.

**Ex. 1.** The HCF of $48\,z^4$ and $80\,z^5$ is

(a) $16z^4$
(b) $z^4$
(c) 16
(d) None of the above

***Sol.*** *(a)* HCF of 48 and 80 is 16 and HCF of $z^4$ and $z^5$ is $z^4$.
$\therefore$ Required HCF of $48\,z^4$ and $80\,z^5$ is $16\,z^4$.

**Ex. 2.** The HCF of $x^3y^2z^5$ and $x^2y^4z^3$ is

(a) $x^3y^3z^3$
(b) $x^2y^2z^2$
(c) $x^2y^2z^3$
(d) $xyz^2$

***Sol.*** *(c)* HCF of $x^3$ and $x^2$ is $x^2$, HCF of $y^2$ and $y^4$ is $y^2$ and HCF of $z^5$ and $z^3$ is $z^3$.
$\therefore$ Required HCF of given monomials is $x^2y^2z^3$.

## HCF of Polynomials

**Case I** Which can be Easily Factorised

*Step* I Resolve both the polynomials in the complete factor form.

*Step* II Required HCF is the product of the HCF of numerical coefficients of the polynomial and factors with their highest powers common to the polynomials.

**Ex. 3.** The HCF of $22x\,(x+1)^2$ and $36x^2(2x^2+3x+1)$ is

(a) $2x(x+1)$ (b) $x(x+1)$ (c) $2(x+1)$ (d) $2(x+1)^2$

***Sol.*** *(a)* Here, HCF of 22 and 36 is 2.
Now, $x(x+1)^2 = x(x+1)(x+1)$
$x^2(2x^2+3x+1) = x^2(2x+1)(x+1)$
Common factors of $x(x+1)^2$ and $x^2(2x^2+3x+1)$ are $x(x+1)$.
Hence, required HCF $= 2(x+1)x$

**Ex. 4.** The HCF of $p(x) = 24(6x^4 - x^3 - 2x^2)$ and $q(x) = 20(2x^6 + 3x^5 + x^4)$ is

(a) $4(2x+1)$ (b) $x^2(2x+1)$
(c) $4x^2$ (d) None of these

***Sol.*** *(d)* Here, HCF of 24 and 20 is 4.
Now, $6x^4 - x^3 - 2x^2 = x^2[6x^2 - x - 2]$
$= x^2[6x^2 - 4x + 3x - 2] = x^2(2x+1)(3x-2)$
and $2x^6 + 3x^5 + x^4 = x^4(2x^2 + 3x + 1)$
$= x^4(2x^2 + 2x + x + 1) = x^4(2x+1)(x+1)$
$\therefore$ Common factors of $6x^4 - x^3 - 2x^2$ and $2x^6 + 3x^5 + x^4$ are $x^2(2x+1)$.
$\therefore$ Required HCF $= 4x^2(2x+1)$

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**Case II** **Which cannot be Easily Factorized**
(Successive Division Method)

*Step* I Find the HCF of the monomial factors, if any of the given polynomials.

*Step* II Arrange the polynomials in ascending or descending order.

*Step* III Divide the polynomial of higher degree by the polynomial of lower degree.

or If both the polynomials are of the same degree any one of them can be taken as divisor or dividend.

*Step* IV After the first division take the remainder as the new divisor and first divisor as new dividend.

*Step* V Continue this process of dividing the last divisor by the last remainder until the remainder becomes zero.

*Step* VI Last divisor is the HCF of the polynomial.

**Point to Remerbered**

- *If the first term of a remainder is negative at any stage, the sign of all of its term must be changed.*
- *If at any stage, the remainder contains common factor take it out.*
- *If the quotient exists as a fraction at any stage, then multiply the dividend by a suitable number to avoid fractional quotients.*

**Ex. 5.** The HCF of $22x^6 - 78x^5 - 16x^2$ and $2x^5 - 78x^2 - 44x$ is

(a) $(x^2 - 3x - 2)$ (b) $2x(x^2 - 3x - 2)$

(c) $22x(x^2 - 3x - 2)$ (d) None of these

**Sol.** *(b)* Here,

$$22x^6 - 78x^5 - 16x^2 = 2x^2(11x^4 - 39x^3 - 8)$$
$$2x^5 - 78x^2 - 44x = 2x(x^4 - 39x - 22)$$

To find HCF of $11x^4 - 39x^3 - 8$ and $x^4 - 39x - 22$

$$\begin{array}{r|l} x^4 - 39x - 22 & 11x^4 - 39x^3 - 8 \ (11 \\ & 11x^4 - 242 - 429x \\ & - \quad + \quad + \\ \hline & -39x^3 + 429x + 234 \end{array}$$

–39 is taken out as common factor from remainder.

$$\begin{array}{r|l} x^3 - 11x - 6 & x^4 - 39x - 22 \ (x \\ & x^4 - 11x^2 - 6x \\ & - \quad + \quad + \\ \hline & 11x^2 - 33x - 22 \end{array}$$

Again 11 is taken out as common fadctor from remainder

$$\begin{array}{r|l} x^2 - 3x - 2 & x^3 - 11x - 6 \ (x + 3 \\ & x^3 - 3x^2 - 2x \\ & - \quad + \quad + \\ \hline & 3x^2 - 9x - 6 \\ & 3x^2 - 9x - 6 \\ & - \quad + \quad + \\ \hline & \times \end{array}$$

$\therefore$ HCF of $11x^4 - 39x^3 - 8$ and $x^4 - 39x - 22$ is $x^2 - 3x - 2$.

Also, HCF of $2x^2$ and $2x$ is $2x$.

$\therefore$ Required HCF $= 2x(x^2 - 3x - 2)$

# Least Common Multiple (LCM)

The lowest common multiple (LCM) of two or more algebraical expressions is the expression of the lowest dimensions which is divisible by each of them without remainder.

## LCM of Monomials

To find the LCM of two monomials, we multiply the LCM of the numerical coefficient of the monomials by all the factors raised to the highest power which it has in either of the given polynomials.

**Ex. 6.** The LCM of $12x^2y^3z^2$ and $18x^4y^2z^3$ is

(a) $x^4y^3z^3$

(b) $9x^2y^2z^3$

(c) $36x^3y^3z^4$

(d) $36x^4y^3z^3$

**Sol.** *(d)* Here, $12x^2y^3z^2 = 2^2 \times 3^1 \times x^2 \times y^3 \times z^2$

$18x^4y^2z^3 = 2^1 \times 3^2 \times x^4 \times y^2 \times z^3$

$\therefore$ Required LCM $= 2^2 \times 3^2 \times x^4 \times y^3 \times z^3$

$= 36x^4y^3z^3$

## LCM of Polynomials

(which can be Easily Factorized)

We resolve both the polynomials as the product of different factors.

Required LCM is the product of the LCM of numerical coefficients of both the polynomials and factors with their highest power which it has in either of the given polynomials.

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**Ex. 7.** The LCM of $x^3 - 2x^2 - x + 2$ and $x^3 - x^2 - 4x + 4$ is

(a) $(x^2 - 1)(x^2 - 4)$ (b) $(x^2 + 2)(x^2 - 4)$

(c) $(x^2 + 1)(x^2 + 2)$ (d) None of these

***Sol.*** *(a)* Here, $x^3 - 2x^2 - x + 2 = x^2(x - 2) - 1(x - 2)$

$= (x - 2)(x^2 - 1) = (x - 2)(x - 1)(x + 1)$

$= (x - 1)(x + 1)(x - 2)$

and $x^3 - x^2 - 4x + 4 = x^2(x - 1) - 4(x - 1)$

$= (x - 1)(x^2 - 4) = (x - 1)(x - 2)(x + 2)$

$\therefore$ Required LCM $= (x - 1)(x + 1)(x - 2)(x + 2)$

$= (x^2 - 1)(x^2 - 2^2)$

$[\because (a - b)(a + b) = a^2 - b^2]$

$= (x^2 - 1)(x^2 - 4)$

**Ex. 8.** The LCM of $a^2 - b^2 - c^2 - 2bc$, $b^2 - c^2 - a^2 - 2ac$ and $c^2 - a^2 - b^2 - 2ab$ is

(a) $(a + b + c)$

(b) $(a - b - c)(a + b + c)$

(c) $(a + b + c)(c - a - b)$

(d) $(a + b + c)(a - b - c)(b - c - a)(c - a - b)$

***Sol.*** *(d)* $a^2 - b^2 - c^2 - 2bc = a^2 - (b^2 + c^2 + 2bc)$

$= a^2 - (b + c)^2 = (a + b + c)(a - b - c)$

$b^2 - c^2 - a^2 - 2ac = b^2 - (c^2 + a^2 + 2ac)$

$= b^2 - (c + a)^2 = (b - c - a)(b + c + a)$

$= (a + b + c)(b - c - a)$

$c^2 - a^2 - b^2 - 2ab = c^2 - (a^2 + b^2 + 2ab)$

$= c^2 - (a + b)^2$

$= (c - a - b)(c + a + b)$

$= (a + b + c)(c - a - b)$

$\therefore$ Required LCM $= (a + b + c)(a - b - c)$

$(b - c - a)(c - a - b)$

**Points to be Remembered**

- *LCM of two polynomials* $= \dfrac{\text{Product of two polynomials}}{\text{HCF of two polynomials}}$
- *HCF of two polynomials* $= \dfrac{\text{Product of polynomials}}{\text{LCM of polynomials}}$
- *For any two polynomials $p(x)$ and $q(x)$*

  $p(x) \times q(x) =$ (Their HCF) $\times$ (Their LCM)

  *i.e. Product of the polynomials*

  = Product of their HCF and LCM

**Ex. 9.** The HCF of two polynomials is $x + 3$ and their LCM is $x^3 - 7x + 6$. If one of the polynomials is $x^2 + 2x - 3$. Then, the other is

(a) $x^2 - x - 6$

(b) $x^2 - x + 6$

(c) $x^2 + x + 6$

(d) None of the above

***Sol.*** *(d)* Here, HCF $= x + 3$

and LCM $= x^3 - 7x + 6$

Also, one polynomial $= x^2 + 2x - 3$

As, LCM $\times$ HCF = Product of two polynomials

Other polynomial $= \dfrac{\text{LCM} \times \text{HCF}}{\text{One polynomial}}$

$= \dfrac{(x + 3) \times (x^3 - 7x + 6)}{x^2 + 2x - 3}$

$= \dfrac{(x + 3)(x^3 - 7x + 6)}{(x + 3)(x - 1)}$

$= \dfrac{(x + 3)(x - 1)(x^2 + x - 6)}{(x + 3)(x - 1)}$

$= x^2 + x - 6$

So, other polynomial is $x^2 + x - 6$.

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