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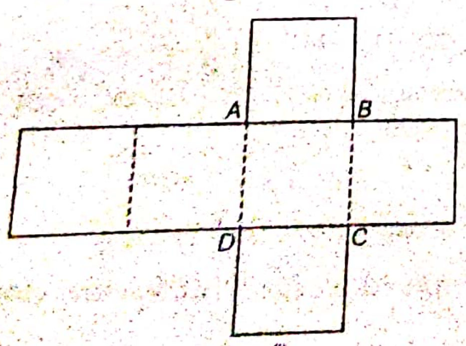
Surface Area and Volume of Solids

In this chapter, we will study to find the surface area and volume of solid figure, like parallelopiped, cube, cuboid, cylinder, cone, frustum of cone, sphere and hemisphere and we will study to find the surface area and volume of combination of solid figures combination of two or more different or similar solid figures.

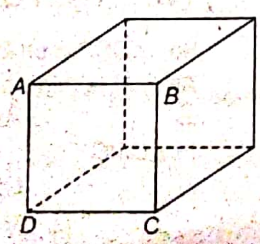
Solid Figures

The objects which occupy space (i.e. they have three dimensions) are called solids. The solid figures can be derived from the plane figures.

e.g. In figure (i), we have a paper cut in the form as shown. It is a plane figure. But when we fold the paper along the dotted lines, a box can be made as shown in figure (ii), which occupies some part of the space. It has more than two dimensions and therefore it fulfils the criteria of being a solid figures.



(i)



(ii)

A solid figure has surface area and volume which are define below :

Surface Area of Solid Figure

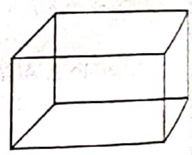
Surface area of a solid body is the area of all of its surfaces together and it is always measured in square units.

Volume of Solid Figure

The measure of part of space occupied by a solid is called its volume. It is always measured in cubic unit.

Parallelopiped

A solid bounded by three pairs of parallel plane surfaces (or faces) is called a parallelopiped. A parallelopiped whose faces are rectangles is called a rectangular parallelopiped or a rectangular solid or a cuboid.

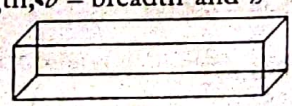


Cuboid

A figure which is surrounded by six rectangular surfaces, is called cuboid.

- A cuboid has 8 corners and 12 edges and four diagonals.
- Volume of cuboid $= l \times b \times h$

  where, l = length, b = breadth and h = height



- Total surface area of cuboid $= 2(lb + bh + lh)$
- Diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$
- Total length of cuboid $= 4(l + b + h)$
- Lateral surface area or Area of 4 walls $= 2(l + b)h$.

Ex. 1. The volume of a cuboid is 880 cm^3, the area of its base is 88 sq cm. Then, its height is

(a) 10 cm (b) 12 cm
(c) 14 cm (d) 16 cm

Sol. (a) Here, volume of a cuboid = 880 cm^3

Area of the base = 88 cm^2

$$\text{Height} = \frac{\text{Volume of the cuboid}}{\text{Area of the base}} = \frac{880}{88} = 10 \text{ cm}$$

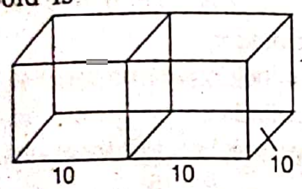
Ex. 2. The length of the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high is

(a) 15 m (b) 16 m
(c) 17 m (d) 17.5 m

Sol. (c) Since, the length of the longest rod is the diagonal of the room

$$= \sqrt{l^2 + b^2 + h^2} = \sqrt{(12)^2 + (9)^2 + (8)^2}$$
$$= \sqrt{144 + 81 + 64} = \sqrt{289} = 17 \text{ m}$$

Ex. 3. Two cubes each of 10 cm edge are joined end-to-end. Then, the surface area of the resulting cuboid is



(a) 100 cm^2 (b) 1000 cm^2
(c) 2000 cm^2 (d) None of these

Sol. (b) Length of the resulting cuboid = 10 + 10 = 20 cm

Breadth of resulting cuboid = 10 cm

Height of resulting cuboid = 10 cm

∴ Surface area of resulting cuboid = $2(lb + bh + hl)$
$= 2[20 \times 10 + 10 \times 10 + 20 \times 10]$
$= 2[200 + 100 + 200]$
$= 2 \times 500 = 1000 \text{ cm}^2$

Ex. 4. The areas of three adjacent faces of a cuboid are x, y and z. If its volume is V, then which is true?

(a) $V = x^3y^2z^2$ (b) $V^2 = xyz$
(c) $V = \sqrt[3]{xyz}$ (d) $V = \frac{x^2y}{z}$

Sol. (b) Let the dimensions of a cuboid be l, b, h respectively, then volume of cuboid, $V = lbh$

Also, $x = lb, y = bh$ and $z = hl$

∴ $V = lbh$

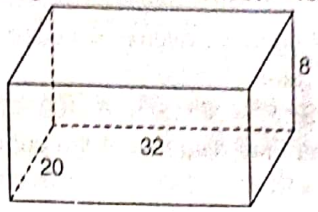
On squaring both sides, we get

$$V^2 = l^2b^2h^2$$
$$= (lb)\cdot(bh)\cdot(hl) = xyz$$

Ex. 5. A metallic sheet is of rectangular shape with dimensions 48 cm × 36 cm from each of its corners a square of 8 cm is cut-off. An open box is made of the remaining sheet, what is the volume of the box?

(a) 13824 cm^3 (b) 1728 cm^3
(c) 5120 cm^3 (d) None of these

Sol. (c) Given length of metallic sheet = 48 cm

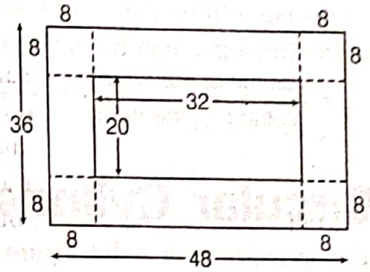


Breadth of metallic sheet = 36 cm

When square of side 8 cm is cut-off from each corner and the flaps turned up, we get an open box whose

Length = 48 − (8 + 8) = 32 cm

Breadth = 36 − (8 + 8) = 20 cm



and height = 8 cm

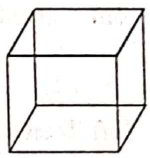
∴ Volume of the box = $l \times b \times h = 32 \times 20 \times 8 = 5120 \text{ cm}^3$

Cube

A cuboid whose length, bradth and height are same is called cube.

- A cube has six surfaces, twelve edges, eight corners, four diagonals.

Cube is a special case of cuboid which has 6 faces of equal length.



- Total length of cube = 12 × side
- Volume of a cube = $(\text{Side})^3$
- Total surface area of a cube = $6 \times (\text{Side})^2$

Diagonal of a cube = $\sqrt{3} \times$ Edge

Ex. 6. If the surface area of a cube is 96 sq cm, then its volume is

(a) 16 cu cm (b) 64 cu cm
(c) 12 cu cm (d) 32 cu cm

Sol. (b) Surface area of cube = $6 \times a^2$, where a = Side of a cube

⇒ $6a^2 = 96 \Rightarrow a^2 = 16 \Rightarrow a = 4$ cm

∴ Volume of the cube = $a^3 = 4^3 = 64$ cu cm

Ex. 7. The surface area and the length of the diagonal of the cube, if the volume of a cube is 2197 cu cm, are

(a) 1012 sq cm and 21.516 cm
(b) 1024 sq cm and 24.516 cm
(c) 1014 sq cm and 22.516 cm
(d) None of the above

Sol. **(c)** Volume of cube = $(\text{Side})^3 = 2197$ cu cm

$\therefore$ Side of cube $= \sqrt[3]{\text{volume of a cube}} = 13$ cm

$\therefore$ Surface area of cube

$= 6\,(\text{Side})^2 = 6\,(13)^2 = 6 \times 169 = 1014$ sq cm

and length of the diagonal of the cube $= \sqrt{3} \times \text{Side}$

$= \sqrt{3} \times 13 = 1.732 \times 13 = 22.516$ cm

Ex. 8. How many 6 m cubes can be cut from a cuboid measuring 36 m × 15 m × 8 m?

(a) 10 (b) 15 (c) 19 (d) 20

Sol. **(d)** Volume of given cuboid $= (36 \times 15 \times 8)\,\text{m}^3$

Volume of the cube to be cut $= (6 \times 6 \times 6)\,\text{m}^3$

$\therefore$ Number of cubes that can be cut from the cuboid

$$= \frac{\text{Volume of the cuboid}}{\text{Volume of the cube}} = \frac{36 \times 15 \times 8}{6 \times 6 \times 6} = 20$$

Right Circular Cylinder

A right circular cylinder is a solid figure obtained by revolving the rectangle, say *ABCD* abouts its one side, say *BC*.

Let base radius of right circular cylinder be *r* and its height be *h*.

Then,

(*i*) Volume of the cylinder

$= (\text{Area of base}) \times \text{Height}$

$= \pi r^2 h$ cu units

(*ii*) Curved surface area = Circumference of the base × Height

$= 2\pi rh$ sq units

(*iii*) Total surface area = Curved surface + Area of two ends

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r)$ sq units

Also, $h = \dfrac{V}{\pi r^2} \Rightarrow r = \sqrt{\dfrac{V}{\pi h}}$

Ex. 9. The volume of a cylinder is $448\,\pi\,\text{cm}^3$ and height 7 cm. Then, its lateral surface area and total surface area are

(a) $349\,\text{cm}^2$ and $753.286\,\text{cm}^2$
(b) $352\,\text{cm}^2$ and $754.286\,\text{cm}^2$
(c) $353\,\text{cm}^2$ and $755.286\,\text{cm}^2$
(d) None of the above

Sol. **(b)** Given, volume of the cylinder $= 448\,\pi\,\text{cm}^3$ and height of the cylinder = 7 cm

Let radius be *r*, then

$\pi r^2 h = 448\,\pi$

$\therefore \quad r^2 = \dfrac{448\,\pi}{h\,\pi} = \dfrac{448}{7} = 64 \Rightarrow r = 8$ cm

$\therefore$ Lateral surface area of the cylinder $= 2\pi rh$

$= 2 \times \dfrac{22}{7} \times 8 \times 7 = 352\,\text{cm}^2$

Total surface area of the cylinder $= 2\pi r\,(h + r)$

$= 2 \times \dfrac{22}{7} \times 8\,(7 + 8)$

$= 2 \times \dfrac{22}{7} \times 8 \times 15 = \dfrac{5280}{7} = 754.286\,\text{cm}^2$

Ex. 10. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Then the ratio of their volumes is

(a) 4 : 9 (b) 16 : 25
(c) 20 : 27 (d) None of these

Sol. **(c)** For first cylinder,

Let radius $= r_1$, height $= h_1$, volume $= V_1$

For second cylinder,

Let radius $= r_2$, height $= h_2$ and volume $= V_2$

Then, $\dfrac{r_1}{r_2} = \dfrac{2}{3}$ and $\dfrac{h_1}{h_2} = \dfrac{5}{3} \Rightarrow r_1 = \dfrac{2r_2}{3}$ and $h_1 = \dfrac{5h_2}{3}$

Required ratio of their volumes be $V_1 : V_2$.

$\Rightarrow \quad \pi r_1^2 h_1 : \pi r_2^2 h_2 \Rightarrow r_1^2 h_1 : r_2^2 h_2$

$\Rightarrow \quad \dfrac{4}{9} r_2^2 \dfrac{5h_2}{3} : r_2^2 h_2 \Rightarrow \dfrac{20}{27} : 1 \Rightarrow 20 : 27$

Ex. 11. A cylindrical bucket of diameter 28 cm and height 12 cm is full of water. The water is emptied into a rectangular tub of length 66 cm and breadth 28 cm. The height to which water rises in the tub is

(a) 2 cm (b) 4 cm
(c) 6 cm (d) 8 cm

Sol. **(b)** Volume of water in the bucket

$= \pi r^2 h = \dfrac{22}{7} \times 14 \times 14 \times 12 = 7392$ cu cm

Let *h* be the height to which water rises in the tub.

$\therefore$ Volume of water in the tub $= 66 \times 28 \times h$ cu cm

According to the question,

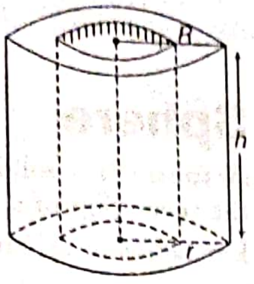
$66 \times 28 \times h = 7392$

$\Rightarrow \quad h = \dfrac{7392}{66 \times 28} = 4$ cm

Hence, the height to wihich water rises in the tub is 4 cm.

Hollow Cylinder

The volume of material in a hollow cylinder is the difference between the volume of a cylinder having the external dimensions and the volume of a cylinder having the internal dimensions.



Let R and r be the external and internal radii of the hollow cylinder and h be its height. Then,

(i) Volume of hollow cylinder $= \pi(R^2 - r^2)\,h$ cu units

(ii) Total surface area $= 2\pi(R+r)(h+R-r)$ sq units

(iii) Curved surface area
$= 2\pi Rh + 2\pi rh = 2\pi(R+r)\,h$ sq units

(iv) Total outer surface area
$= 2\pi rh + \pi R^2 + \pi(R^2 - r^2)$ sq units

Ex. 12. A hollow cylindrical tube, open at both ends is made of iron 1 cm thick. The volume of iron used in making the tube, if the external diameter is 12 cm and the length of tube is 70 cm, is

(a) 2420 cm^3 (b) 2520 cm^3
(c) 2720 cm^3 (d) 2900 cm^3

Sol. **(a)** Here, external radius $(R_1) = 6$ cm

and internal radius $(R_2) = 6 - 1 = 5$ cm

height of hollow cylinder $(h) = 70$ cm

$\therefore$ Volume of iron used in making tube

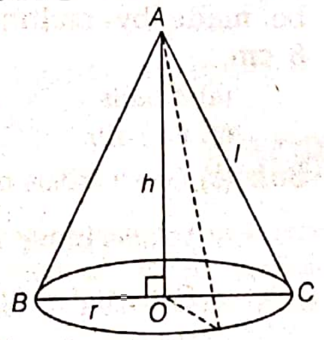
$=$ External volume $-$ Internal volume

$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$

$= \frac{22}{7} \times 70 \times (36 - 25) = 220 \times 11 = 2420$ cu cm

Right Circular Cone

A right circular cone is a solid, generated by the revolution of a right angled triangle about one of its sides containing the right angle as axis. Let height of a right circular cone be h, slant height be l and its radius be r, Then,



(i) The slant height of the cone,

$l = AC = \sqrt{r^2 + h^2}$ units

(ii) Volume of cone $= \frac{1}{3}\pi r^2 h$ cu units

(iii) Curved surface area of cone $= \pi rl$ sq units

(iv) Total surface area of a cone $=$ Curved surface area $+$ Base area $= \pi r(l + r)$ sq units

Ex. 13. The radius and vertical height of a cone are 5 cm and 12 cm, respectively. Then, its lateral surface area is

(a) 202 cm^2 (b) 203.1 cm^2
(c) 204.2 cm^2 (d) 204.3 cm^2

Sol. **(d)** Given, radius of cone $(r) = 5$ cm

Height of cone $(h) = 12$ cm

$\therefore$ Slant height $(l) = \sqrt{r^2 + h^2}$

$= \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ cm

$\therefore$ Lateral/curved surface area $= \pi rl$

$= \frac{22}{7} \times 5 \times 13$

$= \frac{1430}{7} = 204.3$ cm^2

Ex. 14. How many metres of cloth 5 m wide will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m?

(a) 100 m (b) 105 m
(c) 109 m (d) 110 m

Sol. **(d)** Given radius of base $(r) = 7$ m

and vertical height of tent $(h) = 24$ m

Slant height of the tent (l)

$= \sqrt{r^2 + h^2} = \sqrt{24^2 + 7^2} = \sqrt{625} = 25$ m

Now, curved surface area $= \pi rl$

$= \frac{22}{7} \times 7 \times 25 = 550$ m^2

$\because$ Width of cloth $= 5$ m

$\therefore$ Length required to make conical tent

$= \frac{550}{5} = 110$ m

Ex. 15. The radius and the height of a right circular cone are in the ratio 5 : 12. If its volume is 314 m^3, the slant height and the radius are

(a) 12 m, 5 m (b) 13 m, 4 m
(c) 1 m, 4 m (d) 13 m, 5 m

Sol. **(d)** Let radius of cone be $5x$ and the height of the cone be $12x$.

$\therefore$ Volume of cone $= 314 = \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3} \times 3.14 \times (5x)^2 \times (12x) = 314$

$\Rightarrow 314 = 314x^3$

$\Rightarrow x^3 = 1$

$\Rightarrow x = 1$

$\therefore$ Radius $= 5$ m and height $= 12$ m

$\therefore$ Slant height $= \sqrt{r^2 + h^2}$

$= \sqrt{25 + 144} = \sqrt{169} = 13$ m

Hence, the slant height and the radius are 5 m and 13 m.

Ex. 16. The diameters of two cones are equal. If their slant heights are in the ratio 5 : 4. The ratio of their curved surface areas is

(a) 4 : 5 (b) 3 : 5 (c) 5 : 3 (d) 5 : 4

Sol. (d) As diameters are equal,

$\Rightarrow$ Radii are also equal.

Let r be radius of each cone and let slant height of cone be $5x$ and $4x$.

$\therefore$ Curved surface area of first cone $\pi rl = \pi r \times 5x$

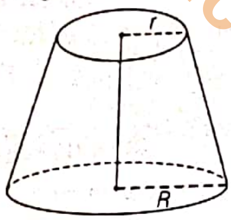
Curved surface area of second cone $\pi rl = \pi r \times 4x$

$\therefore$ The required ratio $= \frac{\pi r \times 5x}{\pi r \times 4x} = 5 : 4$

Frustum of a Cone

If a right circular cone is cut-off by a plane parallel to the base of the cone, then the portion of the cone between the cutting plane and base of the cone is called the frustum of the cone.

Let R, r be the radii of base and top of the frustum of a cone. Let h be the height, then



(i) Volume of frustum of right circular cone

$= \frac{\pi h}{3}[R^2 + r^2 + Rr]$ cu units

(ii) Curved (lateral) surface area of frustum of right circular cone $= \pi l(R + r)$ sq units

where, slant height, $l^2 = h^2 + (R - r)^2$ units

(iii) Total surface area of frustum of right circular cone

= Area of base + Area of top + Lateral surface area

$= \pi[R^2 + r^2 + l(R + r)]$ sq units

(iv) Total surface Area of bucket $= \pi[(R + r)\, l + r^2]$ sq unit

[$\because$ it is open at the bigger end]

Ex. 17. The radii of the ends of a bucket of height 24 cm are 15 cm and 5 cm. Then, its capacity is

(a) 8000 cm^3 (b) 8100 cm^3
(c) 8171.43 cm^3 (d) 8200.43 cm^3

Sol. (c) Given, $h = 24$ cm, $R = 15$ cm and $r = 5$ cm

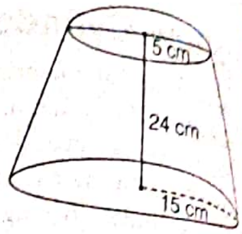
Capacity of bucket = Volume of frustum of a cone

$= \frac{\pi h}{3}[R^2 + r^2 + Rr]$

$= \frac{22}{7} \times \frac{24}{3}[(15)^2 + 5^2 + 15 \times 5]$

$= \frac{22}{7} \times 8[225 + 25 + 75]$

$= \frac{176}{7}(325) = 8171.43\ cm^3$



Sphere

A sphere is a solid generated by the revolution of a semi-circle about its diameter.

Let radius of sphere be r, then

(i) Volume of sphere $= \frac{4}{3}\pi r^3$ cu units

(ii) Surface area of sphere $= 4\pi r^2$ sq units

(iii) Volume of a hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$ cu units

where, r = Inner radius and R = Outer radius

Ex. 18. Given that the volume of a metal sphere is 38808 cm^3. Then, its radius and its surface area are

(a) 7 cm and 616 cm^2
(b) 21 cm and 5544 cm^2
(c) 14 cm and 2464 cm^2
(d) None of the above

Sol. (b) Given, volume of the metal sphere = 38808 cm^3

But volume of sphere $= \frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = 38808$

$\Rightarrow r^3 = 38808 \times \frac{3}{4} \times \frac{7}{22} = 9261$

$\Rightarrow r = (9261)^{1/3} = 21$ cm

$\therefore$ Surface area of the sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 21 \times 21$

$= 5544\ cm^2$

Ex. 19. How many balls each of radius 2 cm can be made by melting a big ball whose radius is 8 cm.

(a) 4 balls (b) 16 balls
(c) 64 balls (d) 128 balls

Sol. (c) Given radius of big ball $(R) = 8$ cm

$\therefore$ Volume of big ball $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(8)^3$

Radius of small ball $(r) = 2$ cm

$\therefore$ Volume of small balls $= \frac{4}{3}\pi(2)^3$

$\therefore$ Required number of balls $= \frac{\text{Volume of big ball}}{\text{Volume of small ball}}$

$= \frac{\frac{4}{3}\pi(8)^3}{\frac{4}{3}\pi(2)^3} = \frac{8 \times 8 \times 8}{2 \times 2 \times 2} = 64$ balls

Ex. 20. A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Then, the length of the wire is

(a) 243 m (b) 343 m (c) 443 m (d) 972 m

Sol. (a) Let the length of wire be h cm.

Volume of sphere = Volume of wire [by condition]

$\Rightarrow \left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right) = \left(\pi \times \frac{2}{10} \times \frac{2}{10} \times h\right)$

$\Rightarrow \frac{h}{25} = 972 \Rightarrow h = (972 \times 25)$ cm

$= \frac{972 \times 25}{100} = 243$ m

Hence, the length of the wire is 243 m.

Ex. 21. If the number of square centimetres on the surface of a sphere is equal to the number of cubic centimetre in its volume. What is the diameter of the sphere?

(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

Sol. (d) Let the radius of sphere be r cm.

Volume of a sphere $= \frac{4}{3}\pi r^3$

$\therefore$ Surface area $= 4\pi r^2$

$\Rightarrow 4\pi r^2 = \frac{4}{3}\pi r^3$ [by condition]

$\Rightarrow r = 3$

$\therefore$ Diameter of the sphere $= 3 \times 2 = 6$ cm

Hemisphere

A plane passing through the centre cuts the sphere in two equal parts, each part is called a hemisphere.

Let the radius of hemisphere be r, then

(i) Volume of hemisphere $= \frac{2}{3}\pi r^3$ cu units

(ii) Curved surface area of hemisphere $= 2\pi r^2$ sq units

(iii) Total surface area $= 2\pi r^2 + \pi r^2 = 3\pi r^2$ sq units

Ex. 22. The volume of two hemisphere are in the ratio 8 : 27. What is the ratio of their radii?

(a) 2 : 3
(b) 3 : 2
(c) 1 : 2
(d) 2 : 1

Sol. (a) Let volumes be V_1 and V_2.

$\therefore V_1 : V_2 = 8 : 27$

$\Rightarrow \frac{2}{3}\pi r_1^3 : \frac{2}{3}\pi r_2^3 = 8 : 27$

$\Rightarrow r_1^3 : r_2^3 = 8 : 27$

$\Rightarrow r_1 : r_2 = 2 : 3$

Important Formulae

- If the side of a cube is increased by x%, then its volume increased by $\left[\left(1 + \frac{x}{100}\right)^3 - 1\right] \times 100\%$.
- If the length, breadth and height of cuboid are made x, y and z times respectively, its volume is increased by $(xyz - 1) \times 100\%$.
- If the length, breadth and height of a cuboid are increased by x%, y% and z% respectively, then its volume is increased by $\left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2}\right]\%$.
- If the sides and diagonal of a cuboid are given, then the total surface area in terms of diagonal and sides is given by
- Total surface area = (Sum of the sides)^2 − (Diagonal)^2.
- If the side of a cube is increased by x%, the surface area is increased by $\left(2x + \frac{x^2}{100}\right)\%$.
- If each side of a cube is doubled, its volume becomes 8 times, i.e. volume is increased by 700%.

Exercise

1. Three equal cubes are placed adjacently in a row. The ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes is

(a) 3 : 1 (b) 6 : 5 (c) 7 : 9 (d) 6 : 7

2. A class room is 7 m long, 6.5 m wide and 4 m high. It has one door 3 m × 1.4 m and three windows each measuring 2 m × 1 m. The interior walls are to be coloured washed. The contractor charges ₹ 5.25 per sq m. The cost of colour washing is

(a) ₹ 519.45 (b) ₹ 159.45 (c) ₹ 513.45 (d) ₹ 419.45

3. The dimensions of a field are 12 m × 10 m. A pit 5 m long, 4 m wide and 2 m deep is dug in one corner of the field and the Earth removed has been evenly spread over the remaining area of the field. The level of the field is raised by

(a) 30 cm (b) 35 cm (c) 38 cm (d) 40 cm

4. A cube of 9 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of base are 15 cm and 12 cm. Then, the rise in water level in the vessel is

(a) 4.05 cm (b) 4 cm
(c) 3.5 cm (d) 3 cm