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Statistics

Statistics is the branch of Mathematics which deals with the collection, analysis and interpretation of numerical data.

In this chapter, we shall extend the study of measures of central tendency i.e. mean, median and mode from ungrouped data to that of grouped data. Concept of cumulative frequency, the cumulative frequency distribution and how to draw cumulative frequency curves (ogive) will also be discussed.

Collection of Data

Collection of data is the first step in statistics towards achieving the goal on conclusion.

On the basis of collection data are of two types

1. **Primary data** The data collected actually in the process of investigation by the investigator. It is original and first hand information is called primary data.
2. **Secondary data** The data collected by someone and used by any other person known as secondary data.

Presentation of Data

Raw or Ungrouped data When the data are presented in random and is not prepared according to some order. It does not give us a clear picture of the class.

Grouped data When the data is arranged in any manner like ascending or descending etc. It can also be presented in the form of a table called frequency distribution table.

Class Intervals Class intervals are the groups in which all the observations are divided. Each class is bounded by two figures (numbers) which are called class limits. The figure on the left side of a class, is called its lower limit and that on the right side of a class, is called its upper limit.

Types of Frequency Distribution

Class marks It is the mid-point of the class interval.

$$\text{Class mark} = \frac{\text{Lower class limit} + \text{Upper class limit}}{2}$$

Range or a class size Difference between the upper lim and the lower limit of a class is called its class size.

Range = Upper limit − Lower limit

e.g. Range of the observations 4, 7, 18, 10, 12

$= 12 - 4 = 8$

Frequency of an observation The number of times a observation occurs is called its frequency.

Frequency distribution of an ungrouped data

The tabular arrangement of data, showing the frequency of each observation is called a frequency distribution.

Types of frequency distribution

There are two types of frequency distribution which is follows:

1. **Discrete frequency distribution** A frequen distribution is called a discrete frequency distribution, data are presented in a way that exact measurements the units are clearly shown.

Marks	Number of students (frequency)
40	7
60	3
80	3
	2
Total	15

2. **Exclusive form** (continuous form) A frequen distribution in which upper limit of each class excluded and lower limit is included, is called exclusive form.

e.g. In this method, the uper limit of a class not included in the class. There, in the class 0-10 of marks obtained by students, a student who has obtained 10 marks is not included in this class. He is counted in the next class 10-20.

3. Inclusive method (discontinous form) In this method,the classes are so formed the uper limit of a class is included in that class. The following example illustrates the method. In class 1000-1099, we include workers having wages between ₹ 1000 and ₹ 1099. If the income of a worker is exactly ₹ 1100, then he is included in the next class 1100-1199.

Exclusive method		Inclusive method	
Wages (in ₹)	Number of workers	Wages (in ₹)	Number of works
1000-1100	125	1000-1099	125
1100-1200	150	1100-1199	150
1200-1300	200	1200-1299	200
1300-1400	250	1300-1399	250
1400-1500	175	1400-1499	175
1500-1600	100	1500-1599	100
Total	1000	Total	1000

It is clear from the above example that both the inclusive and exclusive methods give us the some class frequency although the class intervals are aparently different in the two cases.

In the above example on inclusive method, the difference between the lower limit of a class and upper limit of the preceding class is 1 i.e. $h = 1$.

Therefore, we subtract $\frac{1}{2}$ from the lower limit i.e. $\frac{h}{2}$ and add upper limit of each class $\frac{1}{2}$ to make it continuous.

The adjusted classes would be as follows:

Wages (in ₹)	Number of workers
999.5 - 1099.5	125
1099.5 - 1199.5	150
1199.5 - 1299.5	200
1299.5 - 1399.5	250
1399.5 - 149.5	175
1499.5 - 1599.5	100

Cumulative frequency If the frequency of first class interval is added to the frequency of second class and this sum is added to third class and so on, then frequencies so obtained are known as cumulative frequency.

Classification of Data

There are two methods of classification of data which is as follows :

1. **Exclusive method** When the class intervals are so fixed that the uper limit of one classis the lower limit of the next class it is known as the exclusive method of classification.

Ex. 1. Consider the table given below :

Marks	Number of students (frequency)	Cumulative frequency
0-10	13	13
10-20	7	20
20-30	5	25
30-40	4	29
40-50	1	30
50-60	7	37
60-70	3	40
70-80	4	44
80-90	5	49
90-100	1	50
Total	50	

Then, find the value of the following

(*i*) frequency of class 10-20

(*ii*) class size

(*iii*) mid value of 60-70

(*iv*) total frequencies

(a) 10, 65, 50, 60 (b) 7, 10, 65, 60

(c) 50, 65, 10, 10 (d) 7, 10, 65, 50

Sol. **(d)** (i) Here, frequency of class 10-20 is 7.

(ii) Class size = Upper limit − Lower limit $= 30 - 20 = 10$

(iii) Mid value $= \frac{\text{Upper limit + Lower limit}}{2} = \frac{60+70}{2} = 65$

(iv) Total frequencies = 50

Ex. 2. The class mark of the interval 12.5-17.5 is

(a) 5 (b) 12.5 (c) 15 (d) 17.5

Sol. **(c)** Class mark $= \frac{\text{Lower limit + Upper limit}}{2}$

$$= \frac{12.5+17.5}{2} = \frac{30}{2} = 15$$

Ex. 3. The class marks of a distribution are 54, 64, 74, 84, 94 and 104. Then, the class size is

(a) 5 (b) 10 (c) 54 (d) 104

Sol. **(b)** Since, class size is the difference between the class marks of two adjacent classes.

∴ Class size $= 64 - 54 = 10$

Measures of Central Tendency

An average or central value of a statistical series is the value of the variable which describes the charateristic of the entire distribution. The following are the five measure of central tendency

1. Mathematical averages
   (i) Arithmetic mean or mean
   (ii) Geometric mean
   (iii) Harmonic mean
2. Positional averages
   (i) Median
   (ii) Mode

Out of these measures of central tendency. Arithmetic mean, median and mode are sometimes knows as measures of location.

Arithmetic Mean

The mean (or average) of a number of observations is the sum of the values of all the observations divided by the total number of observations.

Mean = Sum of observations/Number of observation

1. **Arithmetic mean of ungrouped or individual observations** If $x_1, x_2, x_3, \dots, x_n$ are n observations, then

(a) Mean $(\bar{x}) = \dfrac{x_1 + x_2 + \dots + x_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$

It is also called direct method.

(b) Mean $(\bar{x}) = A + \dfrac{1}{n}\sum_{i=1}^{n} d_i$

where, A = Assumed mean and $d_i = x_i - A$.

It is also called shortcut method.

Properties of Arithmetic Mean

(i) If every observation is increased by a constant, then the mean of the observations, so obtained also increases by the same constant.

(ii) If every observation is decreased by a constant, then the mean of the observation, so obtained also decreases by the same constant.

(iii) If each observation is multiplied by a constant, then the mean of the resulting observations can be obtained by multiplying the mean by the same constant.

(iv) If each observation is divided by a constant, then the mean of the resulting observation can be obtained by dividing the mean by the same constant.

(v) The average of odd numbers from 1 to n is $\left[\dfrac{\text{Last odd number}+1}{2}\right]$ and the average of even numbers from 1 to n is $\left[\dfrac{\text{Last even number}+2}{2}\right]$.

Ex. 4. If the heights of 5 persons are 144 cm, 152 cm, 150 cm, 158 cm and 155 cm, respectively. Find the mean height.

(a) 152.5 cm (b) 150 cm (c) 149.8 cm (d) 151.8 cm

Sol. (d) Mean height $= \dfrac{\text{Sum of the heights}}{\text{Number of persons}}$

$= \dfrac{144+152+150+158+155}{5}$

$= \dfrac{759}{5} = 151.8$ cm

Ex. 5. If the average of 6, 8, 5, 7, x and 4 is 7. Then, the value of x is

(a) 10 (b) 11 (c) 12 (d) 15

Sol. (c) $\because$ Sum of observation $= 6+8+5+7+x+4 = 30+x$

and number of observation = 6

$\therefore$ Average $= \dfrac{30+x}{6} \Rightarrow 7 = \dfrac{30+x}{6}$

$\Rightarrow 42 = 30 + x \Rightarrow x = 42 - 30 = 12$

Hence, the value of x is 12.

Ex. 6. The average of 27 observations is 35. If 5 is added to each observation, what will be the new mean?

(a) 10 (b) 20 (c) 30 (d) 40

Sol. (d) Given, $\bar{x} = 35$ and $n = 27$

Sum of observation $= n\bar{x} = 27 \times 35 = 945$

and new total of observation $= 945 + 27 \times 5 = 1080$

$\therefore$ New mean $= \dfrac{1080}{27} = 40$

Shortcut Method

New mean = Previous mean + Number added to each term $= 35 + 5 = 40$

Ex. 7. The mean of 15 observations is 20. If 8 is subtracted from each observation. Then, the new mean is

(a) 10 (b) 12 (c) 16 (d) 20

Sol. (b) Given, mean = 20 and $n = 15$

$\therefore$ Sum of observation $= 15 \times 20 = 300$

and new total of observation $= 300 - 8 \times 15$

$= 300 - 120 = 180$

$\therefore$ New mean $= \dfrac{180}{15} = 12$

Shortcut Method

New mean = Previous mean − Number subtracted from each term $= 20 - 8 = 12$

Ex. 8. The mean of 53 observations is 18. If each observation is multiplied by 3. What will be the new mean?

(a) 18 (b) 36 (c) 53 (d) 54

Sol. (d) Given, $\bar{x} = 18$ and $n = 53$

So, sum of observation $= n\bar{x} = 53 \times 18 = 954$

New total of observation $= 954 \times 3 = 2842$

$\therefore$ New mean $= \frac{2862}{53} = 54$

Shortcut Method

New mean = Previous mean × Constant multiplied to each term $= 18 \times 3 = 54$

2. **Mean of grouped or continuous observations** If $x_1, x_2, x_3, \ldots, x_n$ are n observations whose corresponding frequencies are $f_1, f_2, f_3, \ldots, f_n$, then

(i) **Direct Method**

(a) Mean, $\bar{x} = \frac{x_1 f_1 + x_2 f_2 + \ldots + x_n f_n}{f_1 + f_2 + \ldots + f_n}$

$$= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{\Sigma fixi}{\Sigma fi}$$

(b) Mean, $\bar{x} = A + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i} = A + \frac{\Sigma fidi}{\Sigma fi}$

where, A = Assumed mean and $d_i = x_i - A$.

(ii) **Step Deviation Method** $(\bar{x}) = A + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

where, h = Difference between two consecutive class marks.

Ex. 9. The arithmetic mean of the marks from the following table is

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	12	18	27	20	17	6

(a) 20 (b) 28 (c) 2800 (d) 100

Sol. (b) We have,

Marks	Class mark, x	f	fx
0-10	5	12	60
10-20	15	18	270
20-30	25	27	675
30-40	35	20	700
40-50	45	17	765
50-60	55	6	330
		$\Sigma f = 100$	$\Sigma fx = 2800$

$$\therefore \quad \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{2800}{100} = 28$$

Ex. 10. Find the mean wage from the data given below :

Wage (in ₹)	800	820	860	900	920	980	1000
Number of workers	7	14	19	25	20	10	5

(a) 890 (b) 890.5 (c) 891.2 (d) 100

Sol. (c) Let the assumed mean be $A = 900$.

The given data can be written as under :

Wage (in ₹) (x_i)	Number workers (f_i)	$d = x_i - A$	$f_i d_i$
800	7	−100	−700
820	14	−80	−1120
860	19	−40	−760
900	25	0	0
920	20	20	400
980	10	80	800
1000	5	100	500
Total	$\Sigma f_i = 100$		$\Sigma f_i d_i = -880$

Here, $A = 900$

$\therefore$ Mean $= A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 900 + \frac{-880}{100}$

$= 900 - 8.80 = 891.2$

$\therefore$ Mean wage = ₹891.2

Ex. 11. Find the mean of the following data, by step deviation method.

x	15	25	35	45	55	65
f	4	28	15	20	17	16

Sol. Here, the values of class marks x_i are given, so let $A = 35$ and class width, h = difference between two consecutive class marks $= 10$, then

x_i	f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
15	4	$\frac{15-35}{10} = \frac{-20}{10} = -2$	$4 \times (-2) = -8$
25	28	$\frac{25-35}{10} = \frac{-10}{10} = -1$	$28 \times (-1) = -28$
35 = (a)	15	$\frac{35-35}{10} = \frac{0}{10} = -0$	$15 \times 0 = 0$
45	20	$\frac{45-35}{10} = \frac{10}{10} = 1$	$20 \times 1 = 20$
55	17	$\frac{55-35}{10} = \frac{20}{10} = 2$	$17 \times 2 = 34$
65	16	$\frac{65-35}{10} = \frac{30}{10} = 3$	$16 \times 3 = 48$
Total	$\Sigma f_i = 100$		$\Sigma f_i u_i = 66$

Thus, we have $\Sigma f_i u_i = 66$, $\Sigma f_i = 100$, $h = 10$

$$\therefore \quad \bar{x} = A + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 35 + 10 \times \frac{66}{100}$$

$$= 35 + \frac{660}{100} = 35 + 6.6$$

$$\Rightarrow \quad \bar{x} = 41.6$$

Weighted Arithmetic Mean

If corresponding weight of $x_1, x_2, \dots, x_n$ are $w_1, w_2, \dots, w_n$ respectively, then

Weighted arithmetic mean

$$= \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Ex. 12. The weighted arithmetic mean of the first n natural numbers, the weights being the corresponding numbers, is

(a) $\frac{n+1}{2}$ (b) $\frac{n+2}{2}$

(c) $\frac{2n+1}{3}$ (d) None of these

Sol. *(c)* First n natural numbers are $1, 2, 3, \dots, n$; whose corresponding weights are $1, 2, 3, \dots, n$, respectively.

$$\left[\begin{array}{l} \because \Sigma n^2 = \frac{n(n+1)(2n+1)}{6} \\ \Sigma n = \frac{n(n+1)}{2} \end{array}\right]$$

$$\therefore \text{ Weight arithmetic mean} = \frac{1 \times 1 + 2 \times 2 + \dots + n \times n}{1 + 2 + \dots + n}$$

$$= \frac{1^2 + 2^2 + \dots + n^2}{1 + 2 + \dots + n}$$

$$= \frac{\frac{n(n+1)(2n+1)}{6n(n+1)}}{2} = \frac{2n+1}{3}$$

Combined Arithmetic Mean

If two sets of observations are given, then the combined mean of both the sets can be calculated by the following formula.

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

where, $\bar{x}$ = Mean of sets of observations

$\bar{x}_1$ = Mean of first set of observations

n_1 = Number of observations in first set

$\bar{x}_2$ = Mean of second set of observations

n_2 = Number of observations in second set

Ex. 13. The average salary of male employees in a firm was ₹ 5200 and that of females was ₹ 4200. The mean salary of all the employees was ₹ 5000. The percentage of male and female employees are, respectively.

(a) 80 and 20 (b) 20 and 80

(c) 60 and 40 (d) 52 and 48

Sol. *(a)* Let $x_1 = 5200$, $x_2 = 4200$ and $\bar{x} = 5000$

Also, we know that, $\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$

$$\Rightarrow 5000(n_1 + n_2) = 5200 n_1 + 4200 n_2 \Rightarrow \frac{n_1}{n_2} = \frac{4}{1}$$

$\therefore$ The percentage of male employees in the firm

$$= \frac{4}{4+1} \times 100 = 80$$

and the percentage of female employees in the firm

$$= \frac{1}{4+1} \times 100 = 20$$

Median

After arranging the given data in an ascending or descending order of magnitude, the value of the middle most observation is called the median of the data.

1. **Median of an individual series** If number of observations is n. Then, arrange the observations in ascending or descending order.

   (a) If n is an odd number, then

   $$\text{Median} = \text{Value of the } \left(\frac{n+1}{2}\right)\text{th observation}$$

   (b) If n is an even number, then

   $$\text{Median} = \frac{\text{Value of the} \left(\frac{n}{2}\right)\text{th observation} + \text{Value of} \left(\frac{n}{2} + 1\right)\text{th observation}}{2}$$

2. **Median of a discrete frequency series** First arrange the data in ascending or descending order and find cumulative frequency. Now, find $\frac{N}{2}$, where $N = \Sigma f_i$. See the cumulative frequency just greater than $\frac{N}{2}$. The corresponding value of x is median.

3. **Median of a continuous series** In this case, the class corresponding to the cumulative frequency just greater than $\frac{N}{2}$ is called the median class and the value of median is obtained by the following formula.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c}{f}\right) \times h$$

where, l = Lower limit of median class

f = Frequency of median class

h = Size of median class

c = Cumulative frequency of class before median class

Ex. 14. From the data given, the median of the average deposit balance of saving for the branch during March 1982 is

Average deposit balance (in ₹)	Number of deposit
0 – 100	26
100-200	68
200-300	145
300-400	242
400-500	188
500-600	65
600-700	16

(a) 356
(b) 300
(c) 56.2
(d) 356.2

Sol. *(d)*

Average deposit balance (in ₹)	f	Cumulative frequency (cf)
Less than 100	26	26
100-200	68	94
200-300	145	239
300-400	242	481
400-500	188	669
500-600	65	734
600-700	16	750

$$\frac{N}{2} = \frac{750}{2} = 375$$

The frequency just greater than 375 is 481.

∴ Median class is 300-400.

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c\right)}{f} \times h$$

$$= 300 + \frac{375 - 239}{242} \times 100$$

$$= 300 + 56.2$$

$$= 356.2$$

Mode

1. **Mode of individual series** The value which is repeated maximum number of times is called mode of the series.

Ex. 15. Find the mode for the following series 2.5, 2.3, 2.2, 2.2 2.4, 2.7, 2.7, 2.5, 2.3, 2.2, 2.6 and 2.2.

(a) 2.2 (b) 2.3
(c) 2.7 (d) 2.6

Sol. *(a)* Arranging the data in the form of a frequency table, we have

Value	Ferequency
2.2	4
2.3	2
2.4	1
2.5	1
2.6	1
2.7	2

We see that, the value 2.2 has the maximum frequency 4. So, the mode for the given series is 2.2.

2. **Mode of a discrete frequency series** In this case, mode is the value of the variate corresponding to the maximum frequency.

Ex. 16. Compute the modal value for the following frequency distribution.

x	95	105	115	125	135	145	155	165	175
f	4	2	18	22	21	19	10	3	2

(a) 115 (b) 125
(c) 22 (d) 120

Sol. *(b)* From the given table, it is clear that 125 has the highest frequency i.e. 22. Hence, modal value of the given frequency distribution is 125.

3. **Mode of a continuous series** The class which has maximum frequency is called modal class or group. The mode is given by the formula,

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

where, l = Lower limit of modal class

h = Size of modal class

f_1 = Frequency of modal class

f_0 = Frequency of the class preceding the modal class

f_2 = Frequency of the class succeeding the modal class

Ex. 17. The mode of the following distribution is

(a) 46 (b) 6.66
(c) 46.67 (d) None of these

Class interval	Frequency
0-10	5
10-20	8
20-30	7
30-40	12
40-50	28
50-60	20
60-70	10
70-80	10

Sol. *(c)* Here, maximum frequency is 28. Thus, the class 40-50 is the modal class. Here, $f_1 = 28, f_0 = 12, f_2 = 20$, $l = 40$ and $h = 10$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \frac{10(28-12)}{(2 \times 28 - 12 - 20)}$$

$$\Rightarrow \quad 40 + \frac{10 \times 16}{24} = 40 + 6.666 = 46.67 \text{ (approx)}$$

- Relation among mean, median and mode
  Mode = 3 (Median) − 2 (Mean)
- There is an emprical relationship between the three measures of central tendency, which is given by

Ex. 18. If in a frequency distribution, the mean and median are 20 and 21 respectively, then its mode is approximate by

(a) 24 (b) 23
(c) 25 (d) None of these

Sol. *(b)* Here, mean = 20 and median = 21

Mode = 3 × Median − 2 × Mean
= 63 − 40 = 23

Some Useful Formuale

If $x_1, x_2, \ldots, x_n$ are n observations with their mean M, then deviation i_1 is given by $i_1 = |x_i - M|$.

- Mean deviation for individual series is given by $\frac{\Sigma|\delta_i|}{n}$.
- Mean deviation for discrete series is given by $\frac{\Sigma f|\delta|}{n}$.
- Standard deviation (σ) is given by $\sqrt{\frac{\Sigma|\delta|^2}{n}}$.
- Coefficient of variation $= \frac{\text{Standard deviation}}{\text{Mean}} \times 100$
- Coefficient of mean deviation $= \frac{\text{Mean deviation}}{\text{Mean or median}} \times 100$

Geometric Mean

If a, G and b are in GP, then the geometric mean between a and b is, $G = \sqrt{ab}$.

Ex. 19. What is the geometrical mean of the variate which takes values 210, 201, 202, 20, 12, 10, 2, 1 and 0?

(a) 10 (b) 9
(c) 8 (d) 0

Sol. *(d)* The given variates are 210, 201, 202, 20, 12, 10, 2, 1 and 0.

$\therefore$ Geometric mean of given variates

$= \sqrt[9]{210 \times 201 \times 202 \times 20 \times 10 \times 2 \times 1 \times 0}$

$= \sqrt[9]{0} = 0$

To Insert n Geometric Means between Two Given Numbers

Let a and b two numbers, there can be n geometric means $G_1, G_2, \ldots, G_n$ such that $a, G_1, G_2, \ldots, G_n, b$ form a GP.

In general, $G_k = a\left(\frac{b}{a}\right)^{k/(n+1)}, \forall k = 1, 2, 3, \ldots, n$

- The geometric mean of n positive numbers $a_1, a_2, a_3, \ldots, a_n$ is GM. $\Rightarrow (a_1 a_2 a_3 \ldots a_n)^{1/n}$

Harmonic Mean

The harmonic mean of n positive numbers $a_1, a_2, \ldots, a_n$ is

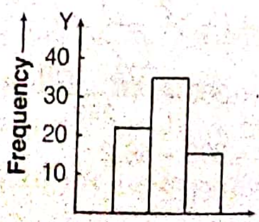
$$\frac{1}{H} = \frac{1}{n}\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_n}\right)$$

where, H denotes the harmonic mean.

Graphical Representation of Data

There are many ways of picturing a frequency distribution of continuous type.

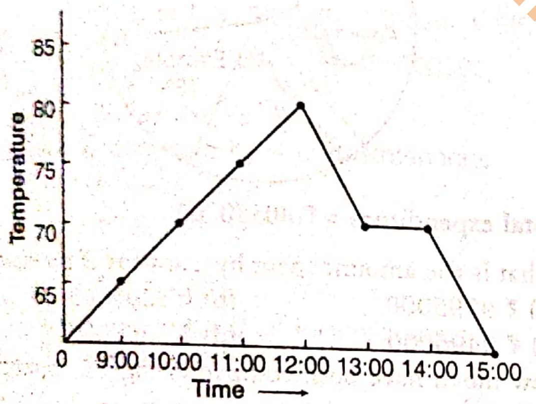
1. **Histogram** A histogram is the graphical representation of a frequency distribution in the form of the rectangles with class intervals as bases, and the corresponding frequencies as height. There being no gap between any two consecutive rectangles.



- A histogram consists of a set of adjacent rectangles, whose bases are equal to class sizes and height are equal to class frequencies
- The total area of the histogram = Sum of areas of all rectangles. If the class intervals are of same size (width), then the area of histogram = NK, where K = Size of a class and N = Sum of frequences of all classes.
- If the frequency distribution is discontinuous (inclusive), change it to continuous (exclusive) and then construct a histogram.

2. **Frequency polygon** A frequency polygon can be drawn joining the mid-points of the respective tops of the rectangle of a histogram in the case of equal class intervals.

A frequency polygon for a grouped data can also be drawn independently by plotting the mid-points of the all classes along X-axis and frequencies along Y-axis and joining the plotted points by straight line. Join the points by free hand.



3. **Ogive (Cumulative Frequency Curve)** When we plot the upper class limits along X-axis and cumulative frequencies along Y-axis, And on joining them we get a curve called an ogive.

We have two types of ogive curves

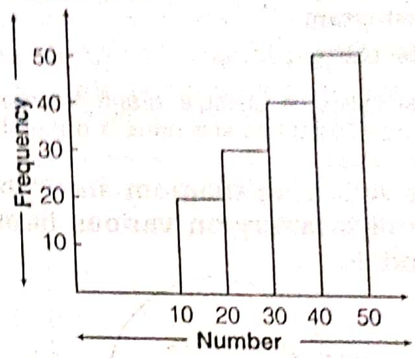
(a) less than ogive (i.e. the rising curve)

(b) more than ogive (i.e. a falling curve)

If we have both ogive (i.e. less than type and more than type), then these two ogives intersect each other at a point. From this point, draw a perpendicular on X-axis, the point at which it cuts the X-axis gives the median. i.e. the x-coordinate of intersection point gives the medium.

Bar Graph

In a bar grah, bars of uniform width are drawn with various heights. The heights of a bar represents the frequency of observation.



Ex. 20. Draw a pie graph representing the below data.

Registraction of vehiclesin 200	Car	Bus	Scooter	Bike
Number of vehicles	20	30	30	40

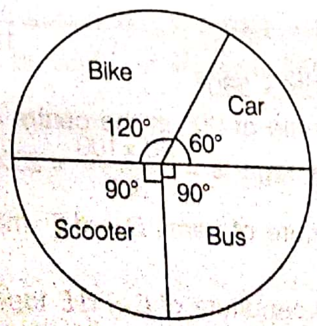
Sol. Divide the central angle i.e. 360° in the proportion of numbers 20, 30, 30, 40.

i.e. 360 should be divided in ratio

20 : 30 : 30 : 40 or 2 : 3 : 3 : 4

Registered vehicle	Number of vehicle	Central angle
Car	20	$\frac{2}{12} \times 360° = 60°$
Bus	30	$\frac{3}{12} \times 360° = 90°$
Scooter	30	$\frac{3}{12} \times 360° = 90°$
Bike	40	$\frac{4}{12} \times 360° = 120°$

The pie chart of the data given below as,

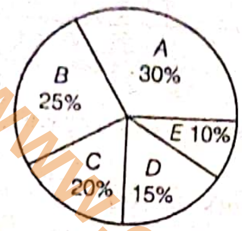


Ex. 21. In statistics, a suitable graph for representing the partitioning of total into subpart is

(a) An ogive
(b) A pictograph
(c) A histogram
(d) A pie chart

Sol. *(d)* In statistics, a suitable graph for representing the partitioning of total into sub parts is a pie chart.

Ex. 22. The following diagram show the expenditure of a family on various items *A*, *B*, *C*, *D* and *E*.



Study the diagram carefully and answer the following questions.

(*i*) The angle of pie diagram showing the expenditure incurred on item *A* is

(a) 30° (b) 35°
(c) 108° (d) None of these

(*ii*) Which two expenditures together will form an angle of 90° at the centre of pie diagram?

(a) *B* and *O* (b) *C* and *A*
(c) *D* and *E* (d) None of these

(*iii*) If the income of the family is ₹ 3000 per month, then expenditure of item *C* will be

(a) ₹ 400 (b) ₹ 500
(c) ₹ 600 (d) ₹ 800

Sol. (i) *(c)* Angle for 100% = 360°

$\therefore$ Angle for expenditure of A = 30% of 360°

$$= \frac{30}{100} \times 360 = 108°$$

(ii) *(c)* $\because$ 100% = 360°

$\therefore$ An angle of 90° at the centre of pie diagram in percentage is $= \frac{90 \times 100}{360} = 25\%$

Expenditure of items *D* and *E* makes upto

$$(15 + 10) = 25\%.$$

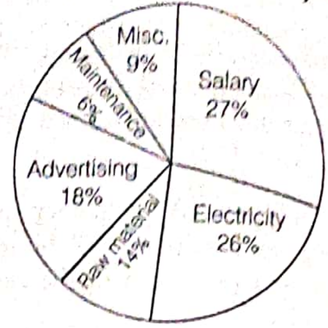
So, expenditure of *D* and *E* together will form an angle of 90° at the centre of pie diagram.

(iii) *(c)* Expenditure on item *C*

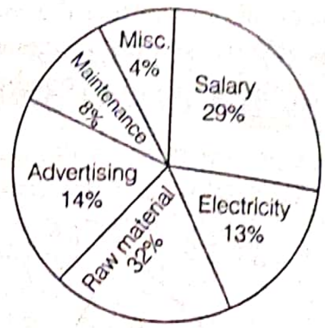
$$= 20\% \text{ of ₹ } 3000 = \frac{20}{100} \times 3000 = ₹\ 600$$

Ex. 23. Study the following graph carefully to answer the questions given below :

Expenditure incurred by two Companies *A* and *B* in a year



Total expenditure = ₹ 35750000



Total expenditure = ₹ 40550000

(*i*) What is the amount spent by company *A* on electricity?

(a) ₹ 9295000 (b) ₹ 9396000
(c) ₹ 9495000 (d) ₹ 9565000

(*ii*) How much have both companies together spent on raw material?

(a) ₹ 15282000 (b) ₹ 17981000
(c) ₹ 15381000 (d) ₹ 17881000

(*iii*) What is the ratio of amount spent by company *A* on advertising and electricity, respectively?

(a) 13 : 9 (b) 14 : 9
(c) 9 : 13 (d) 9 : 14

(*iv*) On which item is there least expenditure? (irrespective of company *A* or *B*)

(a) maintenance of *A*
(b) maintenance of *B*
(c) miscellaneous of *B*
(d) miscellaneous of *A*

(*v*) What is the total amount spent by company *B* on raw material, maintenance and salary together?

(a) ₹ 27979500
(b) ₹ 29779500
(c) ₹ 30979500
(d) ₹ 31333500

(i) (a) Amount spent on electricity by company A

$= 26\%$ of ₹ 35750000

$= ₹\left(35750000 \times \frac{26}{100}\right)$

$= ₹\ 9295000$

(ii) (b) Amount spent by A on raw material

$= 14\%$ of ₹ 35750000

$= ₹\left(35750000 \times \frac{14}{100}\right)$

$= ₹\ 5005000$

Amount spent by B on raw material

$= 32\%$ of ₹ 40550000

$= ₹\left(40550000 \times \frac{32}{100}\right)$

$= ₹\ 12976000$

Required amount $= ₹\ (5005000 + 12976000)$

$= ₹\ 17981000$

(iii) (c) Ratio of amounts spent by A on advertising and electricity

$= (18\%$ of ₹ 35750000) : (26% of ₹ 35750000)

$= \frac{18}{100} \times 35750000 : \frac{26}{100} \times 35750000$

$= 18 : 26 = 9 : 13$

(iv) (c) Expenditure by A on maintenance

$= 6\%$ of ₹ 35750000

$= ₹\left(35750000 \times \frac{6}{100}\right)$

$= ₹\ 2145000$

Expenditure by B on miscellaneous

$= 4\%$ of ₹ 40550000

$= ₹\left(40550000 \times \frac{4}{100}\right)$

$= ₹\ 1622000$

So, it is least on miscellaneous.

(v) (a) Required amount

$= (32 + 8 + 29)\%$ of ₹ 40550000

$= ₹\left(40550000 \times \frac{69}{100}\right)$

$= ₹\ 27979500$

Ex. 24. 70 students from a locality, use different modes of transport to go to school as given below :

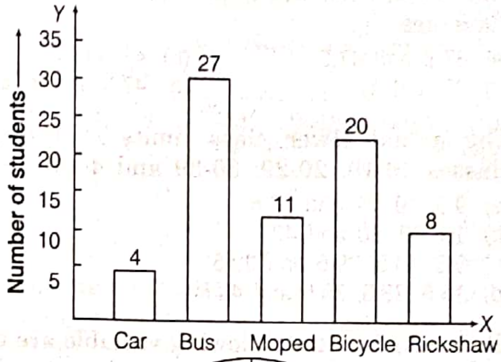
Mode of transport	Car	Bus	Moped	Bicycle	Rickshaw
Number of students	4	27	11	20	8

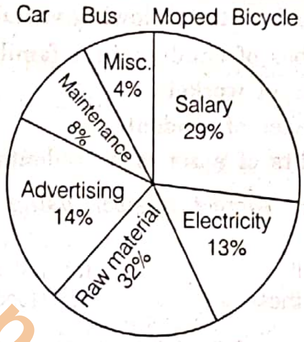
Draw a bar graph representing the above data.

Sol. Take the mode of transport along the X-axis and the number of students along the Y-axis.

All the bars should be of the same width and same space should be left between two consecutive bars.

These bars may be drawn as shown below





Pie Diagram (pie chart)

In a lie graph

- Data is represented by sectors of a circle.
- Each part of data makes a certain central angle.
- Sum of all the angles of sector is 360°.