CLOCK

A clock has two hands: Hour hand and Minute hand. The minute hand (M.H.) is also called the long hand and the hour hand (H.H.) is also called the short hand.

The clock has 12 hours numbered from 1 to 12.

Also, the clock is divided into 60 equal minute divisions. Therefore, each hour number is separated by five minute divisions. Therefore,

Important Points-

- One minute division = = 6° apart, i.e. In one minute, the 60 minute hand moves 6°.
- One hour division = 6° × 5 = 30° apart, i.e. In one hour, the hour hand moves 30° apart.
- Also, in one minute, the hour hand moves = apart.
- Since, in one minute, minute hand moves 6° and hour hand moves, therefore, in one minute, the minute hand gains 5more than hour hand.
- In one hour, the minute hand gains 5 × 60 = 330° over the hour hand. i.e. the minute hand gains 55 minutes divisions over the hour hand.

Relative position of the hands –

- The position of the M.H. relative to the H.H. is said to be the same, whenever the M.H. is separated from the H.H. by the same number of minute divisions and is on same side (clockwise or anticlockwise) of the H.H.
- Any relative position of the hands of a clock is repeated 11 times in every 12 hours.
- When both hands are 15 minute spaces apart, they are at right angle.

- When they are 30 minute spaces apart, they point in opposite directions.
- The hands are in the same straight line when they are coincident or opposite to each other.
- In every hour, both the hand coincide
- In a day, the hands are coinciding 22
- In every 12 hours, the hands of clock coincide 11 times
- In every 12 hours, the hands of clock are in opposite direction 11 times.
- In every 12 hours, the hands of clock are at right angles 22 times.
- In every hour, the two hands are at right angles 2 times.
- In every hour, the two hands are in opposite direction once.
- In a day, the two hands are at right angles 44 times.
- If both the hands coincide, then they will again coincide after 65 minutes e. in correct clock, both hand coincide at an interval of 65 minutes.
- If the two hands coincide in time less than 65 minutes, then clock is too fast and if the two hands coincide in time more than 65 minutes, then the clock is too slow.

NOTE:

ANOTHER SHORT-CUT FORMULA FOR CLOCKS

Angle made by Hands =

Where H = Hour, M = minute

EXAMPLE 1. At what time between 4 and 5 O' Clock will the hands of a watch

(i) Coincide, and

(ii) Point in opposite directions.

Sol.

(i) At 4 O’ clock, the hands are 20 minutes apart. Clearly the minute hand must gain 20 minutes before two hands can be coincident. But the minute-hand gains 55 minutes in 60 minutes.

Let minute hand will gain x minute in 20 minutes.

So,

⇒ min.

∴The hands will be together at min past 4.

(ii) Hands will be opposite to each other when there is a space of 30 minutes between them. This will happen when the minute hand gains (20 + 30) = 50 minutes.

Now, the minute hand gains 50 min in or min.

∴The hands are opposite to each other at min past 4.

EXAMPLE 2. What is the angle between the hour hand and minute hand when it was 5:05 pm.

Sol. 5.05 pm means hour hand was on 5 and minute hand was on 1, i.e. there will be 20 minutes gap.

Angle = 20 × 6°= 120° [∵ 1 minute=6°]

Calendar

Solar year consists of 365 days, 5 hrs 48 minutes, 48 seconds. In 47 BC, Julius Caesar arranged a calendar known as the Julian calendar in which a year was taken as 365 days and in order to get rid of the odd quarter of a day, an extra day was added once in every fourth year and this was called as leap year or Bissextile. Nowadays, the calendar, which is mostly used, is arranged by Pope Gregory XII and known as Gregorian calendar.

In India, number of calendars were being used till recently. In 1952, the Government adopted the National Calendar based on Saka era with Chaitra as its first month, in an ordinary year, Chaitra 1 falling on March 22 of Gregorian Calendar and in a leap year it falls on March 21.

REMEMBER

- In an ordinary year,

1 year = 365 days = 52 weeks + 1 day

- In a leap year

1 year = 366 days =52 weeks + 2 days

NOTE: First January I A.D was Monday. So we must count days from Sunday.

- 100 years or one century contains 76 ordinary years and 24 leap years.

⇒ [76 ×52 weeks+ 76 odd days]

+ [24×52 weeks+ 24 × 2 odd days]

= (76 + 24) × 52 weeks + (76 + 48) odd days

= 100 × 52 weeks + 124 odd days

= 100 × 52 weeks + (17 × 7 + 5) odd days

= (100 × 52 + 17) weeks+ 5 odd days

∴100 years contain 5 odd days.

Similarly, 200 years contain 3 odd days,

300 years contain 1 odd days,

400 years contain 0 odd days.

Year whose non-zero numbers are multiple of 4 contains no odd days; like 800, 1200, 1600 etc.

The number of odd days in months

The month with 31 days contains (4×7 + 3) i.e. 3 odd days and the month with 30 days contains (4×7+2) i.e. 2 odd days.

EXAMPLE 6.What day of the week was 15th August 1949?

Sol. 15th August 1949 means

1948 complete years + first 7 months of the year 1949 + 15 days of August.

1600 years give no odd days.

300 years give 1 odd day.

48 years give {48 + 12} = 60 = 4 odd days.

[∵ For ordinary years → 48 odd days and for leap year 1 more day (48 ÷ 4) = 12 odd days; 60 = 7 × 8 + 4]

From 1st January to 15th August 1949

Odd days:

January-3

February – 0

March-3

April-2

May – 3

June-2

July-3

August – 1

17 ⇒ 3 odd days.

∴ 15th August 1949→ 1 + 4 + 3 = 8 = 1 odd day.

This means that 15th Aug. fell on 1st day. Therefore, the required day was Monday.

EXAMPLE 7. How many times does the 29th day of the month occur in 400 consecutive years?

Sol. In 400 consecutive years, there are 97 leap years. Hence in 400 corse catire year. February has the 29th day 97 times and the remaining eleven month, have the 29th day of the 400×1100 = 4400 times

∴ The 29the day month occurs (4400+97) or 4497 times.

EXAMPLE 8. Today is 5th February. The day of the week is Tuesday. This is leap year. What will be the day of the week on this date after 5 years after 5 years?

Sol. This is a leap year. So, next 3 years will give one odd day each. Then leap year gives 2 odd days and then again next year give 1 odd day.

Therefore (3+ 2+1) =6 odd days will be there.

Hence the day of the week will be 6 odd day 5 beyond

Tuesday, i.e., it will be Monday.

EXAMPLE 9. What day of the week was 20th June 1837?

Sol. 20th June 1837 means 1836 complete years + firs 5 months of the year 1837 + 20 days of June.

1600 years give no odd days.

200 years give 3 odd days.

36 years give (36 + 9) or 3 odd days.

1836 years give 6 odd days.

From 1st January to 20th June there are 3 odd days.

Odd days:

January	:	3
February	:	0
March	:	3
April	:	2
May	:	3
June	:	6
		17

Therefore, the total number of odd days = (6 + 3) or 2 odd days.

This means that the 20th of June fell on the 2nd day commencing from Monday. Therefore, the required day was Tuesday.

EXAMPLE 10. Prove that the calendar for 1990 will same for 2001 also.

Sol. It is clear that the calendar for 1990 will serve for 2001 if first January of both the years is the same weekdays. For that the number of odd day s between 31st December 1989 and 31st December 2000 must be zero. Odd days are as given below.

Year	1990	1991	1992	1993	1994
Odd days	1	1	(Leap) 2	1	1
1995	1996	1997	1998	1999	2000
1	(Leap) 2	1	1	1	(Leap) 2

Total number of odd days = 14 days = 2 weeks + odd days.