CHAPTER 8

Logarithms

Logarithm

If $a > 0$ and $a \neq 1$, then logarithm of a positive number m is defined as the index x of that power of a which equals to m. i.e.,
$\log_a m = x$ iff $a^x = m$

Properties of Logarithm

Let m and n are positive number such that $a, b, c > 0$ and $a, b, c \neq 1$

(i) $\log_a a = 1$

(ii) $\log_a 1 = 0$

(iii) $\log_b a \cdot \log_a b = 1$

(iv) $\log_c a = \log_b a \cdot \log_c b$

Another form $\log_c a = \dfrac{\log_b a}{\log_b c}$

(v) $\log_a (m \cdot n) = \log_a m + \log_a n$

$\log_a \left(\dfrac{m}{n}\right) = \log_a m - \log_a n$

$\log_a m^n = n \log_a m$

(vi) $\log_{n^q}(m^p) = \dfrac{p}{q} \log_n m$

(vii) $a^{\log_a n} = n$

(viii) If $0 < p < 1$, then $a > b \Rightarrow \log_p a < \log_p b$

(ix) If $p > 1$, then $a > b \Rightarrow \log_p a > \log_p b$

(x) $a^{\log_e x} = x^{\log_e a}$, $x > 0, e > 0$

Characterstic and Mantissa

In $\log_{10} N$ the integral part of N is called the characteristic and decimal part of N is called the Mantissa.

(i) (a) If $N > 1$ the characterstic of $\log_{10} N$ is one less than the number of digits in integral part of N.

(b) If $0 < N < 1$, the characteristic of $\log_{10} N$ is one greater than the number of zeros immediately after the decimal point and the first significant digit and is a negative integer.

(ii) **Insert decimal point in antilog of a number**

(a) When characterstic is n, then insert the decimal point after $(n+1)$th digit.

(b) When characterstic is $\bar{n}$, then insert the decimal point such that the first significant digit is at nth place.

- e is the base of natural logarithm (Napier logarithm)
  $\ln x = \log_e x$
- The $\log_{10} e$ is known as Napierian constant
  $\log_{10} e = 0.4342$
- Log of negative integers are not defined, $\log_e 0$ is not defined.
- Logarithmic function is positive as well as negative but exponential function is always positive.

Example 1. Find the value of $\log_2 64$

(a) 32 (b) 6 (c) 6^2 (d) 5

Sol. (b) $\log_2 64 = \log_2 2^6 = 6 \log_2 2 = 6$ $(\because \log_2 2 = 1)$

Example 2. Find the value of $\log_{27}\left(\dfrac{1}{81}\right)$

(a) $\dfrac{3}{4}$ (b) $\dfrac{4}{3}$

(c) $\dfrac{-3}{4}$ (d) $\dfrac{-4}{3}$

Sol. (d) $\log_{27}\left(\dfrac{1}{81}\right) = \log_{3^3}(3)^{-4}$

$= \dfrac{-4}{3} \log_3 3 = \dfrac{-4}{3}$

Scanned by CamScanner

Example 3. Find the value of

$$\log_a b \times \log_b c \times \log_c d \times \log_d a$$

(a) 0 (b) 1

(c) -1 (d) $\frac{ab}{cd}$

Sol. (b) Given expression can be written as

$$\frac{\log_m b}{\log_m a} \cdot \frac{\log_m c}{\log_m b} \cdot \frac{\log_m d}{\log_m c} \cdot \frac{\log_m a}{\log_m d} = 1$$

Alternate Method. Suppose $\log_a b = x, \log_b c = y, \log_c d = z$ and $\log_d a = u$.

Then $a^x = b, b^y = c, c^z = d$

and $d^u = a$

$\therefore$ $a = d^u = (c^z)^u = c^{zu} = (b^y)^{zu} = b^{yzu}$

$\Rightarrow$ $a = b^{yzu} = (a^x)^{yzu} = a^{xyzu}$

$\Rightarrow$ $a^1 = a^{xyzu}$

$\Rightarrow$ $xyzu = 1$

Hence, $\log_a b \times \log_b c \times \log_c d \times \log_d a = 1$

Example 4. What is the value of $\frac{\log_{27} 9 \times \log_{16} 64}{\log_4 \sqrt{2}}$?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) 8 (d) 4

Sol. (d) $\frac{\log_{27} 9 \times \log_{16} 64}{\log_4 \sqrt{2}} = \frac{\log 9}{\log 27} \times \frac{\log 64}{\log 16} \times \frac{\log 4}{\log \sqrt{2}}$

$$= \frac{2\log 3}{3\log 3} \times \frac{6\log 2}{4\log 2} \times \frac{2\log 2}{\frac{1}{2}\log 2}$$

$$= \frac{2}{3} \times \frac{6}{4} \times 4 = 4$$

Example 5. If $(\log_3 x)^2 + \log_3 x < 2$, then which one of the following is correct?

(a) $0 < x < \frac{1}{9}$ (b) $\frac{1}{9} < x < 3$

(c) $3 < x < \infty$ (d) $\frac{1}{9} \le x \le 3$

Sol. (b) $(\log_3 x)^2 + \log_3 x < 2$

$\Rightarrow$ $(\log_3 x)^2 + (\log_3 x) - 2 < 0$

$\Rightarrow$ $(\log_3 x + 2)(\log_3 x - 1) < 0$

$\Rightarrow$ $-2 < \log_3 x < 1$

$\Rightarrow$ $3^{-2} < x < 3^1$

$\Rightarrow$ $\frac{1}{9} < x < 3$

Example 6. Given, $\log_{10} 2 = 0.3010$, what is the number of digits in 20^{64}?

(a) 81 (b) 82

(c) 83 (d) 84

Sol. (d) Let $x = 20^{64}$

Taking log on both sides, we get

$$\log x = 64 \log 20 = 64 \times 1.3010$$

$$= 83.264$$

$\therefore$ Number of digits in $20^{64} = 83 + 1 = 84$

Example 7. For what value(s) of x is

$$\log_{10}(999 + \sqrt{x^2 - 3x + 3}) = 3?$$

(a) 0 (b) 1 only

(c) 2 only (d) 1, 2

Sol. (d) $\log_{10}(999 + \sqrt{x^2 - 3x + 3}) = 3$

$\Rightarrow$ $999 + \sqrt{x^2 - 3x + 3} = 1000$

$\Rightarrow$ $\sqrt{x^2 - 3x + 3} = 1$

$\Rightarrow$ $x^2 - 3x + 3 = 1$

$\Rightarrow$ $x^2 - 3x + 2 = 0$

$\Rightarrow$ $x^2 - 2x - x + 2 = 0$

$\Rightarrow$ $x(x-2) - 1(x-2) = 0$

$\Rightarrow$ $(x-1)(x-2) = 0$

$\Rightarrow$ $x = 1, 2$

Example 8. If $2\log_8 N = p, \log_2 2N = q$ and $q - p = 4$, then find N?

(a) 512 (b) 536

(c) 548 (d) 560

Sol. (a) We are given that

$$2\log_8 N = p \quad \ldots(i)$$

$$\log_2 2N = q \quad \ldots(ii)$$

and $$q - p = 4 \quad \ldots(iii)$$

Now, from Eq. (i), $8^p = N^2 \Rightarrow 2^{3p} = N^2 \quad \ldots(iv)$

From Eq. (ii), $2N = 2^q \Rightarrow N = \frac{2^q}{2} = 2^{q-1} \quad \ldots(v)$

$\therefore$ From Eqs. (iv) and (v), $2^{3p} = (2^{q-1})^2$

$\Rightarrow$ $2^{3p} = 2^{2q-2}$

$\Rightarrow$ $3p = 2q - 2$ ...(vi)

$\Rightarrow$ $2q - 3p = 2$

Now, on solving Eqs. (iii) and (vi), we get $q = 10$

From Eq. (v), $N = 2^{10-1} = 2^9 = 512$

Scanned by CamScanner